ce

CERN LIBRARIES, GENEVA
92.0798.6

IAEA-ICTP 91-TH
du 9/10/91
IC/91/74

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

RELATIVISTIC MODEL
FOR STATEVECTOR REDUCTION

Philip Pearle

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1991 MIRAMARE - TRIESTE

International Atomic Energy Agency
and

United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

RELATIVISTIC MODEL FOR STATEVECTOR REDUCTION *

Philip Pearle

International Centre for Theoretical Physics, Trieste, Italy
and

Hamilton College, Clinton, NY 13323, USA **

MIRAMARE - TRIESTE

April 1991

ABSTRACT

A relativistic quantum field model describing statevector reduction for fermion states is presented. The time evolution of the states is governed by a Schrödinger equation with a Hamiltonian that has a Hermitian and a non-Hermitian part. In addition to the fermions, the Hermitian part describes positive and negative energy mesons of equal mass, analogous to the longitudinal and timelike photons of electromagnetism. The meson-field-sum is coupled to the fermion field. This "dresses" each fermion so that, in the extreme nonrelativistic limit (non-moving fermions), a fermion in a position eigenstate is also in an eigenstate of the meson-field-difference with the Yukawa-potential as eigenvalue. However, the fermions do not interact: this is a theory of free dressed fermions. It is possible to obtain a stationary normalized "vacuum" state which satisfies two conditions analogous to the gauge conditions of electromagnetism (i.e., that the meson-field-difference, as well as its time derivative, give zero when applied to the vacuum state), to any desired degree of accuracy. The non-Hermitian part of the Hamiltonian contains the coupling of the meson-field-difference to an externally imposed c-number fluctuating white noise field, of the CSL (Continuous Spontaneous Localization) form. This causes statevector reduction, as is shown in the extreme nonrelativistic limit. For example, a superposition of spatially separated wavepackets of a fermion will eventually be reduced to a single wavepacket: the meson-field-difference discriminates among the Yukawa-potential "handles" attached to each wavepacket, thereby selecting one wavepacket to survive by the CSL mechanism. Analysis beyond that given in this paper is required to see what happens when the fermions are allowed to move. (It is possible that the "vacuum" state becomes involved in the dynamics so that the "gauge" conditions can no longer be maintained.) It is shown how to incorporate these ideas into quantum electrodynamics. It is proposed that the scalar $i(\partial_\mu A_\mu^\dagger - \partial_\mu A_\mu)$ (+ and - refer to the positive and negative frequency parts of A_μ) play the role of the meson-field-difference, and be coupled to the white noise in the CSL way. Gauge invariance is not violated by the reduction process. The square of the Coulomb potential becomes the "handle" by means of which statevector reduction takes place.

* Submitted for publication.

** Permanent address.

1. Introduction

1a. State Vector Reduction

In the last sentence of his last paper on the foundations of quantum theory, John Bell¹ raised "The big question" as to whether the "precise picture" of reality, inherent in theories which give a dynamical description of statevector reduction, "can be redeveloped in a Lorentz invariant way." This paper introduces such a possible redevelopment.

We begin with a rather long introduction, first because the reader may be unfamiliar with statevector reduction theories and second, in order to completely inform the reader in a qualitative way about the full content of the paper.

It is 25 years since the first proposal² was made to modify the Schrödinger equation by adding a term nonlinear in the wavefunction so that it describes statevector reduction, i.e. produces the evolution

$$\alpha_1(0)|a_1\rangle + \alpha_2(0)|a_2\rangle \rightarrow |a_1\rangle \text{ with probability } |\alpha_1(0)|^2 \quad \text{OR} \quad (1.1)$$

$$\rightarrow |a_2\rangle \text{ with probability } |\alpha_2(0)|^2$$

Motivation for such heresy is the belief that the statevector should represent reality. The status of the statevector in present-day quantum theory is unsatisfactory in this regard because it describes the unreal, i.e., superpositions of macroscopically distinguishable states (Schrödinger's "cat paradox"). It is 15 years since it was shown^{3,4} that it is possible to achieve reduction behavior by adding to Schrödinger's equation an extra term depending upon a randomly fluctuating (white noise) function of time. Over the next 10 years, some further understanding was achieved.

The reduction dynamics was seen to be closely analogous to a "gambler's ruin" game,⁵ wherein e.g. two gamblers repeatedly bet a fixed sum on a coin toss, until one gambler loses all his money: the fluctuations of the gambler's money is analogous to the fluctuations of $|\alpha_n(t)|^2$. The quantum result (1.1) has its parallel in the game: for example, two gamblers, starting play with \$70 and \$30, have respectively a 70% and 30% chance of winning. This is because the game is "fair," and so it was seen that the quantum predictions in (1.1) are due to the "fair" or Martingale nature of the dynamics (i.e., $\langle d|\alpha_n(t)|^2/dt \rangle = 0$, where $\langle \cdot \rangle$ refers to the average over the ensemble of possible fluctuation-caused evolutions).

The evolution (1.1) is supposed to occur rapidly when the states $|a_n\rangle$ are "macroscopically distinct" states, such as those describing an apparatus registering differing outcomes of an experiment, but a mechanism to define those states was never

spelled out (the "preferred basis problem"). When the states $|a_n\rangle$ only differ microscopically, the reduction dynamics is supposed to still occur, although slowly, but a mechanism that turns down the reduction rate for a microscopic system (and turns it up for a macroscopic system) was also never spelled out (the "trigger problem"). However, since for a microscopic system, reduction dynamics is different from the usual quantum evolution, it was realized that it is possible to experimentally test^{6,7} these ideas, but the accuracy required to distinguish a reduction theory from quantum theory appears at present to be unachievable.

The importance of constructing reduction theories so that the density matrix describing the ensemble of reducing statevectors obeys a closed evolution equation, in order that the theories not allow superluminal communication, was pointed out by Gisin.⁸⁻¹¹

It was also realized that reduction theories can produce small violations of conservation laws.¹²

But it was not until 5 years ago that a convincing resolution of the preferred basis and trigger problems was found, when Ghirardi, Rimini and Weber introduced their SL (Spontaneous Localization) theory.¹³⁻¹⁶ Their idea was that a physical random process is always present which infrequently (on an average of once every $\lambda^{-1} \approx 10^{16}$ secs. $\approx 10^8$ years) suddenly alters the wavefunction of a particle, narrowing it by multiplying it by a Gaussian function (of width $\alpha^{-1/2} \approx 10^{-5}$ cm., and with a center that statistically favors appearing where the wavefunction is large). This spontaneous localization or "hitting" process solves the trigger problem because, while the hits occur so infrequently that a single particle is scarcely disturbed, a macroscopic system of N particles will frequently (with average rate $N\lambda$) have a particle hit: if the macroscopic system is in a superposition of spatially separated states, one such hit will reduce the whole wavefunction to one of these states. The preferred basis problem is solved: it is essentially the difference in position of a large number of particles that distinguishes one macroscopic state from another.

Although GRW's SL theory was not a theory which utilizes a modified Schrödinger equation, it became possible to incorporate the GRW ideas in a modified Schrödinger equation with an additional term that, remarkably, is linear in the wavefunction.¹⁷ We presently appear to have a reasonable nonrelativistic theory which describes statevector reduction, and moreover we have a framework that permits generalizations.^{18,19} This will be outlined in section 2. Both the nonrelativistic theory as well as the more general framework will be called CSL (Continuous Spontaneous Localization) theories.

The nonrelativistic CSL theory is Galilean invariant in a way that is different from the usual Galilean invariance of quantum theory. In the CSL theory, the

Hamiltonian depends upon a fluctuating (white noise) function of space and time, so there really are an infinite number of possible Hamiltonians, each with an associated probability of occurrence. It is the ensemble of Hamiltonians (and their associated probabilities) which is invariant under Galilean transformations, but each individual Hamiltonian is not Galilean invariant. Since the theoretical description and its predictions are Galilean invariant when their statistical nature is taken into account, we call this stochastic Galilean invariance.²⁰

1b. Stochastic Lorentz Invariant Models

In section 3 we will look at a recent proposal for a stochastic Lorentz invariant theory.²⁰ The nonreducing part of the Schrödinger equation describes a standard quantum field theory, of a fermion field $\psi(x)$ coupled to a scalar meson field $\phi(x)$. The reducing part of the Schrödinger equation couples the meson field to the white noise in the CSL way. Now (as shown in section 2), when a set of commuting operators is coupled to white noise in the CSL way, the dynamics tends to reduce the statevector to an eigenstate of those operators. Thus, in this case, the dynamics tends to reduce the statevector to an eigenstate of $\phi(x)$ at each point of space.

This behavior can be used to reduce a superposition of states containing fermions by the following mechanism. As is well known, because the fermion field is coupled to the meson field, a fermion state is "dressed" with mesons. This is most simply discussed in the extreme nonrelativistic limit (section 3c) where an unmoving single fermion at a point is in an eigenstate of $\phi_+(x)$ with eigenvalue given by the Yukawa-potential at x ($\phi_+(x)$ is the positive frequency part of $\phi(x)$, the part containing annihilation operators of momentum). Thus, for a superposition of states of a fermion at different locations, each state has a "handle," its attached Yukawa-potential. If $\phi(x)$ were to be replaced by $\phi_+(x)$ in the coupling to the white noise, the CSL reducing mechanism would choose a handle and reduce the statevector to the attached fermion at a single location. Even though $\phi(x)$ is not replaced by $\phi_+(x)$ (and we cannot do this without destroying the Lorentz invariance of the theory), it turns out that the statevector is still nicely reduced by this mechanism.

The trouble is that $\phi_-(x)$ also has its effect. Its effect on any state amounts to its effect on the vacuum since it commutes with the operators that create dressed mesons from the vacuum. And, because of the white noise coupling which contains all wavelengths and frequencies in equal amounts, energy is generated from each vacuum mode (associated to the momentum of the meson field) at the same constant rate (section 3b). This results in an infinite meson energy/volume production rate. Thus, although as far as the fermions are concerned, the reduction behavior of this theory is

satisfactory (section 3d), the theory's behavior as a whole is not satisfactory if one wishes eventually to identify the mesons with real particles.

Nevertheless, in anticipation of a more satisfactory model, the relativistic invariance of such a theory and its conceptual consequences with regard to such matters as nonlocality, causality and objective reality, have been explored.²¹⁻²³

There is another example in nature of the dressing of a fermion than that provided by π -mesons dressing a nucleon, and that is given by quantum electrodynamics. In QED, a positive energy field (of longitudinal photons) and a negative energy field (of timelike photons) contrive to vanish like the Cheshire Cat in *Alice in Wonderland*, leaving behind a Smile, the Coulomb potential dressing each fermion and the Coulomb interaction energy.

In section 4a we present a model with analogous features. It contains a positive energy scalar meson field $A(x)$ and a negative energy scalar meson field $B(x)$. The sum field $S(x) \equiv 2^{-1/2} [A(x) + B(x)]$ is coupled to the fermion field of mass M with coupling constant g . In spite of the coupling, the field S turns out to be a free field, and this makes the model exactly soluble. The fermions become dressed, but otherwise remain free particles. In the extreme nonrelativistic limit (section 4c), a fermion at x_1 is in an eigenstate of the difference field $R(x) \equiv 2^{-1/2} [A(x) - B(x)]$ (not just of $R_+(x)$ with eigenvalue $gY(x-x_1)$ (Y is the Yukawa-potential)).

In the beginning of section 4b it is shown that there is a zero average energy production rate in this model, which is a wholesome sign. Next, the "vacuum" state is discussed. The comments at the end of the last paragraph are predicated on the existence of such a suitable normalized "vacuum" state which the dressed fermion creation operators can act upon. We use quotation marks because the "vacuum" is not simply the no-meson state. We are pursuing the analogy to QED, where gauge conditions restrict all states. In particular, in the Coulomb gauge, the vacuum state is not at all empty, as it is restricted to a particular configuration of longitudinal and timelike photons. Similarly, we will consider restricting the "vacuum" $|v\rangle$ by the two "gauge" conditions $R(x)|v\rangle=0$ and $\dot{R}(x)|v\rangle=0$. (Unlike the case in QED, where the Coulomb gauge restriction removes apparent Lorentz invariance, the "gauge" conditions here are Lorentz invariant.) We find that it is possible to construct a normalized no-fermion state that satisfies these conditions to any desired accuracy.

But there are, of course, many other no-fermion states, and we shall call these *all* states of the "vacuum." In particular, there is a fascinating set of time-dependent "vacuum" states which are eigenstates of $R(x)$ at any time, such that the eigenvalues of $R(x)$ as functions of space-time can be regarded as a sample of a white noise field! Moreover, when the reduction mechanism is included, it is to these states of the "vacuum" that a general statevector strives to reduce. (This totally unexpected

appearance of white noise functions in the theory may presage the long sought (by me, anyway) mechanism whereby the externally imposed white noise field can be eliminated in favor of an internally appearing substitute, but I cannot yet see all the needed features.) However, the "gauge" restricted vacuum state undergoing reduction dynamics evolves negligibly, so the "gauge" invariance of $|\nu\rangle$ is effectively maintained.

Sections 4d and 4e conclude our discussion of the field theory without reduction, with consideration of some new aspects that arise when the extreme nonrelativistic limit is successively replaced by the nonrelativistic limit (where it is found that motion of the fermions modifies the yukawa-potential to first order in g over a distance M^{-1} making it finite at the origin) and the full relativistic treatment of the fermions.

In the full relativistic treatment, when constructing the physical fermion "vacuum," it is best to think in terms of Dirac's original picture of positive and negative energy fermions. We construct creation operators (which are dressed by the interaction) for both of these kinds of particles, and then apply, to the no-fermion state, a complete set of negative energy fermion creation operators, in order to build the fermion "vacuum." To first order in g we find that this "vacuum" comes with built-in fluctuations. It is mostly the state of no bare fermions or antifermions but there are small amplitudes for it to contain one or more bare fermion-antifermion pairs.

We also note that even the bare fermion and antifermion are not eigenstates of the g -charge, except at zero velocity where the bare fermion has positive g -charge and the bare antifermion has negative g -charge. At non-zero velocity, the effective Yukawa-potential of a particle has decreased strength. This means that, when reduction is introduced, the reduction rate, of e.g. a moving apparatus, is less than that of an apparatus at rest - but this is what might be expected as a consequence of time-dilation.

In section 4f, we consider statevector reduction when fermions are present. It is shown, in the extreme nonrelativistic limit, how nearly statevector reduction takes place to eigenstates of $R(x)$. In the nonrelativistic limit, a qualitative difference may emerge because the moving fermion interacts with the "vacuum." The "gauge" conditions may become violated, but this complicated set of affairs needs more study than is attempted here. However, it is possible that the "gauge"-restricted vacuum state is an artificial choice for the full theory. When reduction takes place, it may be more natural to think of the "vacuum" as one of an ensemble of time evolving states. The "gauge" conditions can perhaps be maintained in a time-averaged, ensemble-averaged sense. Anyhow, the important question of the energy changes of a moving fermion undergoing reduction dynamics is left unsolved.

In section 4g, the numerical reduction rates for a single fermion and for a macroscopic object are played around with.

One of the great needs of dynamical reduction theories has been to establish a connection with more established physical theories. It has been suggested that statevector reduction is related to gravity.^{25,28,20} However, here we do not attempt to modify gravity by the addition of statevector reduction but instead, in section 5, we propose a possible modification of QED. We suggest that the scalar field $i(\partial_\mu A_\nu - \partial_\nu A_\mu)$ is the appropriate analog to $R(x)$ that may be coupled to the white noise. We show that the QED Hamiltonian for the free timelike and lightlike photons is identical in form to the free Hamiltonian in the model of section 4. (However, the fermion-meson interaction is different.) It is also seen that the reduction process does not violate the true gauge invariance of QED. The basic equations are displayed. However, in this paper, no detailed calculation based on QED with reduction are attempted. Instead, the discussion closes with some speculations leading to the suggestion that the (dimensionless) coupling constant characterizing reduction in this case is $\approx 10^{-40}$, and with an expression for the reduction rate.

2. CSL

Statevector reduction is the consequence of the dynamical equation^{17-20,23,27,29}

$$d|\nu, \nu\rangle/dt = -iH|\nu, \nu\rangle + \sum_n [A_n w_n(t) - \lambda A_n^2] |\nu, \nu\rangle \quad (2.1)$$

together with the probability rule

$$P(|\nu, \nu\rangle) = P_0(w_n(t)) \langle \nu, \nu | \nu, \nu \rangle \quad (2.2)$$

In Eq. (2.1), H is the usual Hamiltonian and $\{A_n\}$ are a set of commuting operators: it is to one or another joint eigenvector of these operators $(\dots a_n \dots \rightarrow$ with $A_n | \dots a_n \dots \rangle = a_n | \dots a_n \dots \rangle$) that the statevector strives to reduce. $w_n(t)$ are independent white noise functions of time. The probability $P(|\nu, \nu\rangle)$ of the statevector $|\nu, \nu\rangle$ is the same as the probability of the white noise function $w_n(t)$ that generates it. If Eq. (2.1) were treated as a usual (Stratonovich) stochastic differential equation, that probability would be the Gaussian function $P_0(\{w_n(t)\})$ that is completely characterized by the drift and diffusion:

$$\langle\langle w_n(t) \rangle\rangle = 0, \quad \langle\langle w_n(t) w_m(t') \rangle\rangle = \lambda \delta_{nm} \delta(t-t') \quad (2.3a,b)$$

($\langle\langle \dots \rangle\rangle$ means "ensemble average.") However, here we instead assign the probability (2.2) to $w_n(t)$ and the statevector $|\nu, \nu\rangle$. Since the sum in Eq. (2.1) involves non-Hermitian operators, a single normalized statevector, at e.g. time 0, evolves into an

ensemble of statevectors with norms that are generally not equal to 1. Eq. (2.2) says that the fluctuations $\{w_n(t)\}$ that are most probable are those that evolve statevectors of large norm (while still not being too improbable with regard to the distribution P_0). It can be shown¹⁸ that Eq. (2.2) is also equivalent to characterizing the drift of the fluctuations by

$$\langle\langle w_n(t) \rangle\rangle = 2\lambda \langle \psi, t A_n | \psi \rangle \quad (2.3a)$$

while the diffusion still satisfies Eq. (2.3b).

Some simple examples for which Eq. (2.1) can be directly integrated show in detail how the reduction mechanism works in the individual case.^{20,23,29} In this paper we will only need to consider the density matrix $D(t)$ which describes the ensemble of statevectors obeying Eqs. (2.1), (2.2) (see earlier references for the derivation):

$$dD(t)/dt = -i[H, D(t)] - \frac{\lambda}{2} \sum_n [A_n, [A_n, D(t)]] \quad (2.4)$$

To see how Eq. (2.4) describes statevector reduction, we set $H=0$, and solve:

$$\langle \dots a_{i_1} \dots | D(t) \dots a_{i_n} \dots \rangle = e^{-(\lambda t/2) \sum_n (a_{i_1}^\dagger a_{i_1}^2 \dots a_{i_n}^\dagger a_{i_n}^2 \dots)} | D(0) \dots a_{i_1} \dots a_{i_n} \dots \rangle \quad (2.5)$$

The off-diagonal elements of the density matrix decay at a rate characterized by λ as well as by the square of the eigenvalue differences: the more different the eigenvalues, the faster is the reduction. An initial statevector $\sum \alpha_i(0) | \dots a_{i_1} \dots \rangle$ evolves into an ensemble of statevectors, with $| \dots a_{i_1} \dots \rangle$ occurring in the ensemble with probability $|\alpha_i(0)|^2$ as promised in Eq. (1.1).³⁰ When H is not set equal to zero, there is competition between the evolution and the reduction, whose joint effect must be analyzed in any particular case.

To apply these considerations to a collection of fermions, in a way that utilizes the GRW reduction mechanism, we choose^{17,18}

$$A_n \rightarrow A(z) \equiv (\alpha/\pi)^{1/4} \int dx e^{-(\alpha/2)(x-z)^2} \xi^\dagger(x) \xi(x) \quad (2.6)$$

$(\xi^\dagger(x))$ is the fermion creation operator at point x , and $\alpha^{-1/2} \approx 10^{-5}$ cm. is the GRW width parameter.) Roughly speaking, $A(z)$'s eigenvalues are the number of particles in a volume $\alpha^{3/2}$ about the point z . The dynamical equation (2.1) can be written in this case as

$$d|\psi, t\rangle/dt = -iH_f |\psi, t\rangle + \int dx \xi^\dagger(x) \xi(x) \tilde{w}(x, t) |\psi, t\rangle - \lambda \int dx dx' \xi^\dagger(x) \xi(x) \xi^\dagger(x') \xi(x') e^{-(\alpha/4)(x-x')^2} |\psi, t\rangle \quad (2.7)$$

where H_f is the usual nonrelativistic fermion Hamiltonian. Instead of writing Eq. (2.7) in terms of the operators $A(z)$ in (2.6) and the white noise characterized by

$$\langle\langle w(x, t) \rangle\rangle = 0, \quad \langle\langle w(x, t) w(x', t') \rangle\rangle = \lambda \delta(x-x') \delta(t-t') \quad (2.8a, b)$$

(which is the natural generalization of Eqs. (2.3)), as we might have done, we have chosen to write Eq. (2.7) in terms of the number density operators $\xi^\dagger(x) \xi(x)$ and the white noise $\tilde{w}(z)$ (constructed from that in (2.8)) which is characterized by

$$\langle\langle \tilde{w}(x, t) \rangle\rangle = 0, \quad \langle\langle \tilde{w}(x, t) \tilde{w}(y, t') \rangle\rangle = \lambda e^{-(\alpha/4)(x-x')^2} \delta(t-t') \quad (2.9)$$

Eq. (2.9) shows that that $\tilde{w}(x, t)$ is white noise in time, but that its fluctuations in space are correlated over a distance $\approx \alpha^{-1/2}$.

The collection of operators $A(z)$ strives to reduce the statevector to an eigenstate of the number density operator $\xi^\dagger(x) \xi(x)$ at each point x . This is evident in the specialization of Eqs. (2.4), (2.5) to this case:

$$dD(t)/dt = -i[H_f, D(t)] - (\lambda/2) \int dz [A(z), [A(z), D(t)]] \quad (2.10)$$

$$\langle a | D(t) | a \rangle = e^{-(\lambda t/2) \int dz [a(z) - a(z)]^2 \langle a | D(0) | a \rangle} \quad (2.11)$$

$(A(z)|a\rangle = a(z)|a\rangle)$. Eq. (2.11) exhibits both the slow reduction rate when $D(t)$ describes a single particle (since $\int dz a(z)^2 = 1$), as well as the rapid reduction rate when $D(t)$ describes a macroscopic system in a superposition of far separated states (where $\int dz a(z) - a(z) = a(z)$, and is large over a large z -volume).

Because wavepackets are narrowed by the reduction mechanism, there is an accompanying small increase in particle energy.^{13,28} We may rewrite Eq. (2.10) for a single free particle in the position representation

$$\begin{aligned} d\langle x | D(t) | x \rangle / dt &= -i(2M)^{-1} (\nabla^2 - V^2) \langle x | D(t) | x \rangle - \\ &\quad - (\lambda/2) (\alpha/\pi)^{1/2} \int dz [e^{-(\alpha/2)(x-z)^2} e^{-(\alpha/2)(x'-z)^2} \langle x | D(t) | x \rangle] \\ &= -i(2M)^{-1} (\nabla^2 - V^2) \langle x | D(t) | x \rangle - \lambda [1 - e^{-(\alpha/4)(x-x')^2}] \langle x | D(t) | x \rangle \end{aligned} \quad (2.12a, b)$$

It is then straightforward to calculate the ensemble average of the rate of increase of kinetic energy

$$d\text{Tr}((p^2/2M)D(t)) = 3\lambda\alpha/4M \quad (2.13)$$

If the parameters λ, α have the numerical values chosen by GRW, this energy increase is at present too small to be observed.

3. Model with One Scalar Field

3a. Stochastic Relativistic Invariance

The density matrix evolution equation (2.10) is Galilean invariant. One may use the usual space-translation rotation and boost generators to show its invariance under these transformations. It also is manifestly time translation invariant, but it cannot be run backward in time indefinitely without the density matrix losing its positive definite nature (so, strictly speaking, we have Galilean semi-group invariance).

However, this is no physical restriction because Eq. (2.9) is supposed to be used by giving an initial density matrix $D(0)$ (which may be pure or not) and running it forward in time. Usually one only describes a limited system with $D(0)$, as one doesn't know or doesn't care about the state of the rest of the universe. If the system is properly chosen, the forward motion in time of $D(t)$ is indifferent to the state of the rest of the universe, but the backward motion is generally not. Even in the usual quantum theory, where one is mathematically permitted to run a statevector or density matrix indefinitely backward in time, if the rest of the universe is not included the result soon bears no relation to reality. So, ordinary quantum theory also, but not so obviously, has the same restriction.

In scattering theory one sets up a pretend universe in the arbitrary past so that one can maintain complete, and not just semi-group, Galilean invariance, but with dynamical reduction theories one must confront reality. That is, in scattering theory, an initial state of particles can be found, and the interaction turned on far in the past (a non-Galilean invariant procedure!) such that the particles arrive near each other at time ≈ 0 in a state that well approximates the actual scattering that is being simulated, as long as the interaction is long range so that the particles are not influenced by it until they meet. But, in a reduction theory, the interaction is between a particle and the local white noise fluctuation. When this is turned on far in the past, each particle starts reducing far in the past, and the particles do not arrive at time ≈ 0 in a state approximating the actual

situation. So, in a reduction theory, one cannot use the artifice of scattering theory to obtain full Galilean invariance.

The Galilean semi-group invariance of the density matrix evolution (2.10) is a consequence of the stochastic Galilean invariance of the modified Schrödinger equation (2.7) for the individual statevector, and the associated probability rule (2.2). In addition to time translation governed by the non-Hermitian generator displayed in (2.7), the usual other generators may be used to transform the statevector to other frames in a consistent manner.²⁰ A single Hamiltonian characterized by a particular sample function $w(\mathbf{x}, t)$ certainly does not look the same from different frames, but the ensemble of such Hamiltonians and the statevector probabilities do look the same from every frame, and this makes the predictions of the theory, as embodied in the density matrix, Galilean invariant.

Likewise, relativistically invariant CSL-type equations can be created. One we will consider here, in sections 3b-3d has the evolution equation

$$d\psi, t \rangle / dt = -i[H_F + H_m + g \int d\mathbf{x} \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) \phi(\mathbf{x})] \psi, t \rangle + \int d\mathbf{x} [\phi(\mathbf{x}) w(\mathbf{x}, t) - \lambda \phi^2(\mathbf{x})] \psi, t \rangle \quad (3.1)$$

where H_F and H_m are the usual free fermion and meson Hamiltonians

$$H_F = \int d\mathbf{x} \bar{\psi}(\mathbf{x}) [-i\gamma \cdot \nabla + M] \psi(\mathbf{x}), \quad H_m = \frac{1}{2} \int d\mathbf{x} [\pi(\mathbf{x})^2 + \nabla \phi(\mathbf{x}) \cdot \nabla \phi(\mathbf{x}) + m^2 \phi(\mathbf{x})^2] \quad (3.2a, b)$$

and the white noise $w(\mathbf{x}, t)$ (a scalar field under Lorentz transformations) is characterized by Eqs. (2.8). The infinitesimal generators can be constructed to transform the statevector to any Lorentz frame (the boost generator is not the usual one, as it possesses appropriate reducing terms, but the translation and rotation generators are as usual) in a consistent manner.²⁰ The ensemble of Hamiltonians looks the same from all frames.

However, the problem of being unable to go backwards in time appears here in aggravated form. If a statevector is given at time t_0 in Lorentz frame #1, it may be evolved forward in time into an ensemble of statevectors in that frame. But no statevector or ensemble can be defined in a boosted frame #2, for that requires what does not exist, knowledge of a statevector or ensemble in the region of spacetime earlier than time t_0 in frame #1. This same problem appears in the usual relativistic quantum theory, but it is swept under the rug when the artifice of scattering theory is employed.

Nonetheless, the theory displays active Lorentz invariance, as it may be used to describe an experiment performed in any Lorentz frame, and it will give the same statistical results in all frames. And, one may write Eq. (3.1) as a Tomonaga-Schwinger equation for the evolution of the statevector defined on an arbitrary

hypersurface, with the proviso that the evolving surfaces do not cross the hypersurface where the statevector is initially defined, and one can show that this hypersurface evolution has the same description from all frames,²¹ which is an aspect of passive Lorentz invariance.

If we knew the true statevector of a unique universe (assumed Minkowskian) in one Lorentz frame for all time, as well as the true fluctuation $w(\mathbf{x},t)$ that brought it about, since we have the generators, we could find that statevector in any Lorentz frame. If events in that universe were rich enough so that its history includes many repeated "experiments" of an appropriate nature, one could in principle examine its behavior from all frames to see to what statistical accuracy Lorentz invariance is satisfied. If the true universe is "typical" (i.e. it exhibits ergodicity such that the time average behavior of an ensemble of repeated almost identical situations shows the same statistical behavior as the CSL ensemble for such a situation), then it may be possible to establish passive Lorentz invariance for the theory, in some sense. These sketchy remarks should be taken to indicate that the problem is deep (involving probabilistic inferences based upon an individual case) and unsolved, and requires examination.

In order that it be possible to construct the infinitesimal generators, or to employ the Tomonaga-Schwinger formalism, it is sufficient that the white noise $w(\mathbf{x},t)$ be locally coupled to a Lorentz scalar, and that the whole non-Hermitian CSL part of the Hamiltonian (e.g. the last integral in Eq. (3.1) commute with itself on spacelike hypersurfaces.^{20,21} The models considered in this paper are of this type.

3b. Vacuum Behavior

As mentioned in the introduction, the fermions described by Eq. (3.1) get dressed by the meson field because of the g -coupling, and a meson field configuration is selected by the reduction process, resulting in a reduced state of the attached fermion. Although this has been discussed elsewhere²⁰ (in the one-dimensional case) we are going to look at it here.

But first, we look at the obvious difficulty with the model. The density matrix evolution equation (2.4), specialized to Eq. (3.1), is

$$dD(t)/dt = -i[H, D(t)] - \frac{\lambda}{2} \int d\mathbf{x} [\phi(\mathbf{x}), [D(t)]] \quad (3.3)$$

($H = H_f + H_m + H_i$). We calculate the rate of increase of $\bar{H}(t) \equiv \text{Tr}\{HD(t)\}$, the ensemble expectation value of the energy:

$$d\bar{H}(t)/dt = -\frac{\lambda}{2} \text{Tr}\{\int d\mathbf{x} [\phi(\mathbf{x}), [D(t)]]\} = \frac{\lambda}{2} \delta(0) \int d\mathbf{x} \quad (3.4)$$

(using $[\phi(\mathbf{x}), H] = [\phi(\mathbf{x}), H_m] = i\pi(\mathbf{x})$, $[\phi(\mathbf{x}), \pi(\mathbf{x})] = i\delta(0)$, and $\text{Tr}\{D(t)\} = 1$). The energy production rate per unit volume is $(\lambda/2)\delta(0)$, certainly larger than the small amount in Eq. (2.13). Fearless in the face of infinity, we trudge on, noting that it is remarkable that *this same infinite value is obtained regardless of the state of the system described by $D(t)$* : surely that requires some explanation.

In order to see how this comes about, let us set $H_f = 0$, $H_i = 0$ and consider the "vacuum" alone. We inquire as to what the reduction process does to each mode of the "vacuum" state. We write the meson field operators $\phi(\mathbf{x})$, $\pi(\mathbf{x})$ in terms of momentum creation and annihilation operators in the standard way:

$$\phi(\mathbf{x}) = (2\pi)^{-3/2} \int d\mathbf{k} (2\varepsilon)^{-1/2} [a(\mathbf{k}) + a^\dagger(-\mathbf{k})] e^{i\mathbf{k}\cdot\mathbf{x}} \quad (3.5a)$$

$$\pi(\mathbf{x}) = (2\pi)^{-3/2} \int d\mathbf{k} (e/2)^{1/2} i[-a(\mathbf{k}) + a^\dagger(-\mathbf{k})] e^{i\mathbf{k}\cdot\mathbf{x}} \quad (3.5b)$$

($\varepsilon(\mathbf{k}) \equiv \sqrt{\mathbf{k}^2 + m^2}$, $[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}')$). Eq. (3.3) becomes:

$$dD(t)/dt = -i \int d\mathbf{k} e a^\dagger(\mathbf{k}) a(\mathbf{k}), D(t) - \frac{\lambda}{2} \int d\mathbf{k} (2\varepsilon)^{-1} [a(\mathbf{k}) + a^\dagger(-\mathbf{k})] [a^\dagger(\mathbf{k}) + a(-\mathbf{k})] D(t) \quad (3.6)$$

We note that the modes $\mathbf{k}$ and $-\mathbf{k}$ are coupled by the reduction interaction term. It is useful to define new operators $q_1(\mathbf{k})$, $q_2(\mathbf{k})$, $p_1(\mathbf{k})$, $p_2(\mathbf{k})$ satisfying

$$[q_1(\mathbf{k}), p_1(\mathbf{k}')] = i\delta_{ij}\delta(\mathbf{k} - \mathbf{k}'), \quad [q_1(\mathbf{k}), q_1(\mathbf{k}')] = [p_1(\mathbf{k}), p_1(\mathbf{k}')] = 0 \quad (3.7)$$

which are related to $a(\mathbf{k})$, $a^\dagger(\mathbf{k})$ by

$$a(\mathbf{k}) + a^\dagger(-\mathbf{k}) \equiv q_1(\mathbf{k}) - iq_2(\mathbf{k}), \quad -ia(\mathbf{k}) + ia^\dagger(-\mathbf{k}) \equiv p_1(\mathbf{k}) - ip_2(\mathbf{k}) \quad (3.8a,b)$$

and which preserve the commutation relations of $a(\mathbf{k})$, $a^\dagger(\mathbf{k})$, as can be deduced from:

$$a(\mathbf{k}) = 2^{-1} [(q_1(\mathbf{k}) + ip_1(\mathbf{k})) - i(q_2(\mathbf{k}) + ip_2(\mathbf{k}))] \quad (3.9a)$$

$$a^\dagger(\mathbf{k}) = 2^{-1} [(q_1(\mathbf{k}) - ip_1(\mathbf{k})) + i(q_2(\mathbf{k}) - ip_2(\mathbf{k}))] \quad (3.9b)$$

(we have used $q_1(-\mathbf{k})=q_1(\mathbf{k})$, $q_2(-\mathbf{k})=q_2(\mathbf{k})$, $p_1(-\mathbf{k})=p_1(\mathbf{k})$, $p_2(-\mathbf{k})=p_2(\mathbf{k})$). We note from Eq. (3.5a) and Eq. (3.8a) that an *eigenbasis* of $q_i(\mathbf{k})$ is also an *eigenbasis* of $\phi(\mathbf{x})$.

Expressing Eq. (3.6) in terms of the new operators, we obtain

$$\begin{aligned} d\tilde{D}(t)/dt = & -i\int d\mathbf{k} \frac{\xi}{2} (q_1^2 + p_1^2 + q_2^2 + p_2^2), D(t) \\ & - (\lambda/2) \int d\mathbf{k} e^{-i\{[q_1, [q_1, D(t)]] + [q_2, [q_2, D(t)]]\}} \end{aligned} \quad (3.10)$$

($\int d\mathbf{k}$ denotes that the integral is to be taken over a suitable half of $\mathbf{k}$ space, so that if $\mathbf{k}$ is included, $-\mathbf{k}$ is not included.)

Eq. (3.10) describes a collection of harmonic oscillators which are being reduced according to their position. Going over to a discrete basis, we make the replacements $q_i(\mathbf{k}) \rightarrow (\Delta k)^{-3/2} q_i$, $p_i(\mathbf{k}) \rightarrow (\Delta k)^{-3/2} p_i$, where $(\Delta k)^3$ is the element of $\mathbf{k}$ -space volume. We consider a density matrix $D(t)$ which can be written as the product of density matrices $\tilde{D}(\mathbf{k}, t)$ for each mode, each of which satisfies

$$d\tilde{D}(t)/dt = -i(\epsilon/2)[q^2 + p^2, \tilde{D}(t)] - (\lambda/2\epsilon)[q, [q, \tilde{D}(t)]] \quad (3.11)$$

This problem, of such a reducing oscillator, was first considered by Diosi³¹ and has the solution

$$\begin{aligned} \langle q | \tilde{D}(t) | q \rangle = & [\pi(1 + \lambda\epsilon^{-1}t - 2^{-1}\epsilon^{-2}\lambda\sin 2\epsilon t)]^{-1/2} \cdot \\ & \cdot e^{-(1/4)(q-q')^2[1 + \lambda\epsilon^{-1}t + 4^{-1}\lambda\epsilon^{-2}\sin 2\epsilon t]} \cdot \\ & \cdot e^{-(1/4)[(q+q') + i\lambda 2^{-1}\epsilon^{-2}(q-q')(1-\cos 2\epsilon t)]^2/[1 + \lambda\epsilon^{-1}t - 2^{-1}\epsilon^{-2}\lambda\sin 2\epsilon t]} \end{aligned} \quad (3.12)$$

corresponding to the initial condition that the oscillator starts in the ground state at $t=0$,

$$\langle q | \tilde{D}(0) | q \rangle = \pi^{-1/2} e^{-(q^2 + q'^2)/2} \quad (3.13)$$

Eq. (3.12) clearly shows that the "vacuum" is so excited by the reduction process that it dramatically departs from the ground state. Reduction evolution toward an eigenstate of q , and therefore to an eigenstate of $\phi(\mathbf{x})$, is apparent in the behavior of the first exponential for large t

$$e^{-(\lambda/4\epsilon)(q-q')^2} \approx \sqrt{4\pi\epsilon\lambda t} \delta(q-q') \quad (3.14)$$

On the diagonal, for large t the density matrix approaches

$$\langle q | \tilde{D}(t) | q \rangle \rightarrow [\pi\lambda t/\epsilon]^{-1/2} e^{-q^2\epsilon/\lambda t} \quad (3.15)$$

It follows that $\tilde{D}(t)$ can crudely be thought of as asymptotically consisting of an ensemble of normalized states, each state being close to an eigenstate of q , with all states having equal probability, for eigenvalues lying between $-\lambda t/\epsilon$ and $\lambda t/\epsilon$.

We are interested in $\bar{H}_{\text{mode}}(t) \equiv \text{Tr}[(\epsilon/2)[p^2 + q^2]D(t)]$, the expectation value of the energy. From Eq. (3.12) one readily finds

$$\text{Tr}\{\frac{\xi}{2}q^2D(t)\} = \frac{1}{4}[e + \lambda t - (\lambda/2\epsilon)\sin 2\epsilon t] \quad (3.16a)$$

$$\text{Tr}\{\frac{\xi}{2}p^2D(t)\} = \frac{1}{4}[e + \lambda t + (\lambda/2\epsilon)\sin 2\epsilon t] \quad (3.16b)$$

$$\bar{H}_{\text{mode}}(t) = \frac{1}{2}[e + \lambda t] \quad (3.16c)$$

Thus we see from Eq. (3.16) that $d\bar{H}_{\text{mode}}(t)/dt = \lambda/2$ for each mode. When we sum over all modes we get

$$\begin{aligned} d\bar{H}_{\text{m}}(t)/dt = & \sum_{\mathbf{k}} (\lambda/2) (\Delta k)^{-3} \sum_{\mathbf{k}} [(\Delta k)^3 \rightarrow (\lambda/2)\delta(\mathbf{k})] d\mathbf{k} \\ = & (\lambda/2)(2\pi)^{-3} \int d\mathbf{x} e^{i\mathbf{0} \cdot \mathbf{x}} \int d\mathbf{k} e^{i\mathbf{k} \cdot \mathbf{0}} = (\lambda/2) \int d\mathbf{x} \delta(\mathbf{0}) \end{aligned} \quad (3.17)$$

Eq. (3.17) shows that the rate of energy increase in Eq. (3.4) can be totally accounted for as due to the excitation by reduction of the "vacuum" modes. We are then led to expect, when particles are added to the picture, that their reduction behavior proceeds in a sensible fashion, although occurring against the background of this unfortunate "vacuum" behavior. That this is indeed the case is suggested in the next section.

3c. Extreme Nonrelativistic Limit without Reduction

We shall make our point by considering Eq. (3.1) in the extreme nonrelativistic limit, where the fermions do not move. For simplicity, we remove H_f entirely, and replace $\bar{\psi}(\mathbf{x})\psi(\mathbf{x})$ in the interaction with $\xi^\dagger(\mathbf{x})\xi(\mathbf{x})$ ($\xi^\dagger(\mathbf{x})$ creates a fermion at the point $\mathbf{x}$). This model without reduction is well known,³² but we will review it anyway,

using methods that will also be employed in subsequent sections, before analyzing its reduction behavior.

Consider the Schrödinger evolution

$$d|\psi, \tau\rangle/d\tau = -i[d\mathbf{x}_2^2/2(\pi^2 + \nabla\phi \cdot \nabla\phi + m^2\phi^2 + 2g\xi^\dagger\xi\phi)]|\psi, \tau\rangle \equiv -iH|\psi, \tau\rangle \quad (3.18)$$

with an initial statevector consisting of bare fermions at $\mathbf{x}_1, \dots, \mathbf{x}_N$ (and the "vacuum" in the ground state):

$$|\psi, 0\rangle = \xi_1^\dagger(\mathbf{x}_1) \dots \xi_N^\dagger(\mathbf{x}_N)|0\rangle \quad (3.19)$$

Writing the solution in the form

$$|\psi, \tau\rangle = [e^{-iH\tau} \xi_1^\dagger(\mathbf{x}_1) e^{iH\tau} \dots [e^{-iH\tau} \xi_N^\dagger(\mathbf{x}_N) e^{iH\tau} \dots e^{-iH\tau}]] |\psi, 0\rangle \quad (3.20)$$

suggests that we calculate

$$\tilde{\xi}_i^\dagger(\mathbf{x}, t) \equiv e^{-iHt} \xi_i^\dagger(\mathbf{x}) e^{iHt} \quad \text{with} \quad \tilde{\xi}_i^\dagger(\mathbf{x}, 0) = \xi_i^\dagger(\mathbf{x}) \quad (3.21a, b)$$

(we already have $e^{-iHt}|0\rangle$). We also will need $\tilde{\phi}, \tilde{\pi}, \tilde{\xi}$ defined similarly,

Using

$$d\tilde{O}_p(\mathbf{x}, t)/dt = -i[H, \tilde{O}_p(\mathbf{x}, t)] \quad (3.22)$$

as well as $H = \tilde{H}$, and the invariance of the equal time commutation and anticommutation relations under this unitary transformation, we obtain (note the similarity to equations for the Heisenberg operators $\tilde{O}_p(\mathbf{x}, t)$):

$$d\tilde{\phi}/dt = -\tilde{\pi}, \quad [\square + m^2]\tilde{\phi} = -g\tilde{\xi}^\dagger\tilde{\xi} \quad (3.23a, b)$$

$$d\tilde{\xi}^\dagger/dt = -ig\tilde{\xi}^\dagger\tilde{\phi}, \quad d\tilde{\xi}/dt = 0 \quad (3.23c, d)$$

Since, according to Eq. (3.23d), $\tilde{\xi}^\dagger(\mathbf{x}, t)\tilde{\xi}(\mathbf{x}, t) = \xi^\dagger(\mathbf{x})\xi(\mathbf{x})$, we can solve Eq. (3.23b) for $\tilde{\phi}(\mathbf{x}, t)$ in terms of operators at $t=0$:

$$\tilde{\phi}(\mathbf{x}, t) = \int d\mathbf{x}' [\phi(\mathbf{x}') \tilde{G}(\mathbf{x}-\mathbf{x}', t) - \pi(\mathbf{x}') G(\mathbf{x}-\mathbf{x}', t)] - g \int_0^t dt' \int d\mathbf{x}' \xi^\dagger(\mathbf{x}') \xi(\mathbf{x}') G(\mathbf{x}-\mathbf{x}', t-t') \quad (3.24)$$

In Eq. (3.24), $G(\mathbf{x}, t)$ is the retarded greens function

$$G(\mathbf{x}, t) = \theta(t) (2\pi)^{-3/2} \int d\mathbf{k} e^{i\mathbf{k} \cdot \mathbf{x}} E^{-1} \sin Et \quad (3.25)$$

satisfying $G(\mathbf{x}, 0) = 0$, $\dot{G}(\mathbf{x}, 0) = \delta(\mathbf{x})\theta(t)$, $[\square + m^2]G(\mathbf{x}, t) = \delta(\mathbf{x})\delta(t)$. Eq. (3.23c) can be integrated:

$$\begin{aligned} \tilde{\xi}_i^\dagger(\mathbf{x}, t) &= e^{ig \int_0^t dt' \int d\mathbf{x}' \phi(\mathbf{x}') \tilde{G}(\mathbf{x}-\mathbf{x}', t-t')} e^{ig \int_0^t dt' \int d\mathbf{x}' \pi(\mathbf{x}') G(\mathbf{x}-\mathbf{x}', t-t')} \\ &\quad \cdot e^{ig^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int d\mathbf{x}' \xi^\dagger(\mathbf{x}') \xi(\mathbf{x}') G(\mathbf{x}-\mathbf{x}', t_1-t_2)} \\ &\quad \cdot e^{-ig^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int d\mathbf{x}' \tilde{G}(\mathbf{x}-\mathbf{x}', t_1) G(\mathbf{x}-\mathbf{x}', t_2) \cdot \xi_1^\dagger(\mathbf{x}')} \end{aligned} \quad (3.26)$$

Eq. (3.26) shows how the bare fermion creation operator is dynamically dressed. Accordingly, to obtain the completely dressed fermion, we consider

$$\tilde{\xi}_i^\dagger(\mathbf{x}) \equiv \lim_{t \rightarrow \infty} e^{-ig^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int d\mathbf{x}' \xi^\dagger(\mathbf{x}') \xi(\mathbf{x}') G(\mathbf{x}-\mathbf{x}', t_1-t_2) \cdot \tilde{\xi}_i^\dagger(\mathbf{x}, t)} \quad (3.27)$$

[An exponential factor in Eq. (3.26) has been taken out because, when applied to the state (3.10), it merely introduces a numerical factor $\exp -iEt$, where E is the Yukawa (infinite) self-energy and interaction energy among the fermions. This would become $\exp -iE_\infty$ in the $t \rightarrow \infty$ limit, were it allowed to remain.] We can perform the time integrals of G and $\tilde{G}$ in Eq. (3.26) using

$$\lim_{t \rightarrow \infty} \int_0^t dt e^{-iEt} \sin Et = P\left(\frac{1}{E}\right), \quad \lim_{t \rightarrow \infty} \int_0^t dt e^{-iEt} \cos Et = \pi \delta(E) \quad (3.28)$$

(the convergence factor $e^{-\delta t}$ in (3.21) can be thought of as replacing g by $ge^{-\delta t}$, i.e., to turning off the interaction in the infinite future), obtaining

$$\int_0^\infty dt \tilde{G}(\mathbf{x}-\mathbf{x}', t) = 0, \quad \int_0^\infty dt G(\mathbf{x}-\mathbf{x}', t) = \frac{e^{-m|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|} \equiv Y(\mathbf{x}-\mathbf{x}') \quad (3.29a, b)$$

Eq. (3.27) then becomes

$$\tilde{\xi}_i^\dagger(\mathbf{x}) = e^{ig \int d\mathbf{x}' \pi(\mathbf{x}') Y(\mathbf{x}-\mathbf{x}') \xi_1^\dagger(\mathbf{x}')} \quad (3.30)$$

Eq. (3.30) is the dressed fermion creation operator. We note by direct calculation that it satisfies

$$[\phi(y), \tilde{\xi}^{\dagger}(x)] = -g y(x-x') \tilde{\xi}^{\dagger}(x) \quad (3.31a)$$

$$[H, \tilde{\xi}^{\dagger}(x)] = \tilde{\xi}^{\dagger}(x) [-g^2/2] d\mathbf{x} \tilde{\xi}^{\dagger}(x) \tilde{\xi}(x) y(x-x') - (g^2/2) y(0) \quad (3.31b)$$

It follows from Eq. (3.31b) that $\tilde{\xi}^{\dagger}(x)$ creates energy eigenstates (in-states):

$$H \tilde{\xi}^{\dagger}(x_1) \dots \tilde{\xi}^{\dagger}(x_N) |0\rangle = [- (g^2/2) \sum_{n,n'=1}^N y(x_n - x_{n'})] \tilde{\xi}^{\dagger}(x_1) \dots \tilde{\xi}^{\dagger}(x_N) |0\rangle \quad (3.32)$$

(the infinite self-energy terms $\sim y(0)$ may be subtracted from H if desired). However, these are not eigenstates of $\phi(y)$, since

$$\begin{aligned} \phi(y) \tilde{\xi}^{\dagger}(x_1) \dots \tilde{\xi}^{\dagger}(x_N) |0\rangle &= [-g \sum_{n=1}^N y(y-x_n)] \tilde{\xi}^{\dagger}(x_1) \dots \tilde{\xi}^{\dagger}(x_N) |0\rangle + \\ &+ \tilde{\xi}^{\dagger}(x_1) \dots \tilde{\xi}^{\dagger}(x_N) \phi(y) |0\rangle \end{aligned} \quad (3.33)$$

although they are eigenstates of $\phi_{\perp}(y)$ with eigenvalue $-g^2 y(y-x_n)$. Nonetheless, the state in Eq. (3.33) is close enough to an eigenstate of $\phi(y)$ so that Eq. (3.3) produces reduction, in this extreme nonrelativistic limit, based upon differences in the Yukawa-potential, as will now be shown.

3d. Extreme Nonrelativistic Limit With Reduction

Consider the density matrix evolution equation (3.3) in the extreme nonrelativistic limit

$$dD(t)/dt = -i[\frac{1}{2} d\mathbf{x} [\pi^2 + \nabla \phi + \pi^2 \phi^2 + 2g^2 \tilde{\xi}^{\dagger} \tilde{\xi} \phi], D(t)] - \frac{\lambda}{2} \int d\mathbf{x} [\phi(x), [\phi(x), D(t)]] \quad (3.34)$$

For simplicity, we will only treat a single fermion in an initial pure state

$$D(0) = \int d\mathbf{x} f(x) \tilde{\xi}^{\dagger}(x) |0\rangle \langle 0| \int d\mathbf{x}' f^*(x') \tilde{\xi}(x') \quad (3.35)$$

$|0\rangle$ is the no-fermion state, and $|0\rangle$ is the ground state of the "vacuum" although the analysis for a many-fermion state goes through similarly. In order to solve Eq. (3.34) we make the ansatz

$$D(t) = \int d\mathbf{x} d\mathbf{x}' f(x) \tilde{\xi}^{\dagger}(x) |0\rangle \langle 0| \int d\mathbf{x}_1 \mathbf{x}'_1 f^*(x'_1) \tilde{\xi}(x'_1) \quad (3.36a)$$

$$D(\mathbf{x}, \mathbf{x}', 0) = |0\rangle \langle 0| \quad (3.36b)$$

so that D is a density matrix solely in the "vacuum" space. Putting Eq. (3.36a) into Eq. (3.34), and utilization of Eqs. (3.31) to commute H and ϕ past $\tilde{\xi}^{\dagger}$, results in the dynamical equation for D :

$$\begin{aligned} \partial D(\mathbf{x}, \mathbf{x}', t)/\partial t &= -i[H_m, D(\mathbf{x}, \mathbf{x}', t)] - (\lambda/2) \int d\mathbf{y} [\phi(y), [\phi(y), D(\mathbf{x}, \mathbf{x}', t)]] \\ &- (\lambda g^2/2) \int d\mathbf{y} [y(x-y) - y(x'-y)]^2 D(\mathbf{x}, \mathbf{x}', t) \\ &- \lambda g \int d\mathbf{y} [y(x-y) - y(x'-y)] [\phi(y), D(\mathbf{x}, \mathbf{x}', t)] \end{aligned} \quad (3.37)$$

If the first two terms on the right hand side of Eq. (3.37) were the only terms, we would be dealing with the "vacuum" reduction analyzed in section 3b.

If the third term on the right hand side of Eq. (3.37) were the only term, the solution of (3.37) would be

$$D(\mathbf{x}, \mathbf{x}', t) = e^{- (\lambda g^2 t/2) \int d\mathbf{y} [y(x-y) - y(x'-y)]^2 f(x) f^*(x') |0\rangle \langle 0|} \quad (3.38)$$

This is identical to the nonrelativistic CSL reduction of a particle described in section 2 (in the $M \rightarrow \infty$ limit where the particles do not move), except that the Gaussians in e.g. Eq. (2.12a) are replaced by Yukawa-potentials.

It is the last term in Eq. (3.37), which describes the interaction of the "vacuum" with the particle, that prevents us from visualizing this process as separate particle and "vacuum" reductions. To see how important this term is, we do a modal analysis of Eq. (3.37), as was performed for the vacuum in section 3b. In terms of the variables $q_1(k), q_2(k), p_1(k), p_2(k)$ introduced there, Eq. (3.37) becomes:

$$\begin{aligned}
\partial D(\mathbf{x}, \mathbf{x}', t) / \partial t = & -i \int d\mathbf{k} \frac{\mathbf{E}}{2} (q_1^2 + p_1^2 + q_2^2 + p_2^2), D^\dagger] - \\
& - (\lambda/2) \int d\mathbf{k} e^{-i\{[q_1, [q_1, D]] + [q_2, [q_2, D]]\}} \\
& - \lambda g^2 (2\pi)^{-3} \int d\mathbf{k} e^{-4\{[\cos \mathbf{k} \cdot \mathbf{x} - \cos \mathbf{k} \cdot \mathbf{x}']^2 + [\sin \mathbf{k} \cdot \mathbf{x} - \sin \mathbf{k} \cdot \mathbf{x}']^2\}} D^\dagger \\
& - \lambda g^2 (2\pi)^{-3} \int d\mathbf{k} e^{-5/2\{[\cos \mathbf{k} \cdot \mathbf{x} - \cos \mathbf{k} \cdot \mathbf{x}'] [q_1, D^\dagger] + [\sin \mathbf{k} \cdot \mathbf{x} - \sin \mathbf{k} \cdot \mathbf{x}'] [q_2, D^\dagger]\}} \quad (3.39)
\end{aligned}$$

As in section 3b we can go over to a discrete basis, and write the density matrix as the product of density matrices $\tilde{D}(\mathbf{x}, \mathbf{x}', t)$ for each mode, each of which satisfies

$$\partial \tilde{D}(\mathbf{x}, \mathbf{x}', t) / \partial t = -i [\frac{\mathbf{E}}{2} (q^2 + p^2), \tilde{D}] - (\lambda/2) [q, [\tilde{D}]] - \lambda_1 \tilde{D} - \lambda_2 [q, \tilde{D}] \quad (3.40)$$

where

$$\lambda_1 = \lambda g^2 (2\pi)^{-3} (\Delta k)^3 e^{-4\{[\cos \mathbf{k} \cdot \mathbf{x} - \cos \mathbf{k} \cdot \mathbf{x}']^2 \text{ OR } [\sin \mathbf{k} \cdot \mathbf{x} - \sin \mathbf{k} \cdot \mathbf{x}']^2\}} \quad (3.41a)$$

$$\lambda_2 = \lambda g^2 (2\pi)^{-3} (\Delta k)^3/2 e^{-5/2\{[\cos \mathbf{k} \cdot \mathbf{x} - \cos \mathbf{k} \cdot \mathbf{x}'] \text{ OR } [\sin \mathbf{k} \cdot \mathbf{x} - \sin \mathbf{k} \cdot \mathbf{x}']\}} \quad (3.41b)$$

(the cosines are used for mode q_1, p_1 , the sines for mode q_2, p_2). The solution of Eq. (3.40), expressed in the basis in which q is diagonal, subject to the initial condition (3.13) that the "vacuum" is in the ground state at $t=0$, is (compare Eq. (3.12))

$$\langle q | \tilde{D}(\mathbf{x}, \mathbf{x}', t) | q \rangle = [\pi(1 + \lambda e^{-1} t - 2^{-1} e^{-2} \lambda \sin 2\epsilon t)]^{-1/2} \cdot e^{-\lambda_1 t} \cdot$$

$$\cdot e^{-(1/4)(q-q')^2 [1 + \lambda e^{-1} t + \lambda_4^{-1} e^{-2} \sin 2\epsilon t]} \cdot e^{-\lambda_2 \epsilon^{-1} (q-q') \sin \epsilon t}.$$

$$\cdot e^{-(1/4)[(q+q') + i\lambda_2^{-1} e^{-2} (q-q')(1 - \cos 2\epsilon t) - 2i\lambda_2 \epsilon^{-1} (1 - \cos \epsilon t)]^2 / [1 + \lambda e^{-1} t - 2^{-1} e^{-2} \lambda \sin 2\epsilon t]} \quad (3.42)$$

The point to observe here is that the interaction terms proportional to λ_2 do nothing more dramatic than oscillate with time. Although the vacuum reduction is somewhat correlated with the fermion reduction, *the essential behavior of the density matrix can be described as the product of the reduction on fermion position described by the λ_1 -term, and the reduction of the vacuum toward eigenstates of $\phi(\mathbf{x})$ (with a long time behavior that is not essentially different from that discussed in section 3b).*

We conclude that reduction of fermion states via the "handle" of an attached Yukawa-potential works in this model, even though the "vacuum" behaves unacceptably. This leads us to seek another model where the same reduction mechanism operates, but with a more physically justifiable "vacuum" behavior.

4. Model with Two Scalar Fields

4a. The Meson Fields

The model that will be considered in the rest of this paper is designed to dress fermions in a way that imitates the behavior of the longitudinal and timelike photons in QED. We consider the following evolution equation for statevectors:

$$d|\psi, t\rangle/dt = -i[H_f + H_m + H_i]|\psi, t\rangle + \int d\mathbf{x} [R(\mathbf{x})w(\mathbf{x}, t) - \lambda R(\mathbf{x})^2]|\psi, t\rangle \quad (4.1a)$$

where H_f is the usual fermion Hamiltonian given in Eq. (3.2a), but

$$H_m = \frac{1}{2} \int d\mathbf{x} [\pi_A^2 + \nabla_A \cdot \nabla_A + m^2 A^2 - \pi_B^2 - \nabla_B \cdot \nabla_B - m^2 B^2] \quad (4.1b)$$

$$H_i = g \int d\mathbf{x} \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) S(\mathbf{x}) \quad (4.1c)$$

$$R(\mathbf{x}) \equiv \frac{1}{\sqrt{2}} [A(\mathbf{x}) - B(\mathbf{x})], \quad S(\mathbf{x}) \equiv \frac{1}{\sqrt{2}} [A(\mathbf{x}) + B(\mathbf{x})] \quad (4.1d,e)$$

That is, according to Eq. (4.1b), A is a positive energy scalar meson field and B is a negative energy scalar meson field. The only nonvanishing commutators of the basic field operators are the usual ones

$$[A(\mathbf{x}), \pi_A(\mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'), \quad [B(\mathbf{x}), \pi_B(\mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}') \quad (4.2a,b)$$

Actually, rather than working with the fields A and B , we prefer to work with the fields R and S and their conjugate momenta

$$\pi_R(\mathbf{x}) \equiv \frac{1}{\sqrt{2}} [\pi_A(\mathbf{x}) - \pi_B(\mathbf{x})], \quad \pi_S(\mathbf{x}) \equiv \frac{1}{\sqrt{2}} [\pi_A(\mathbf{x}) + \pi_B(\mathbf{x})] \quad (4.3a,b)$$

It follows from Eqs. (4.1d,e), (4.2), (4.3) that the only nonvanishing commutators among R, S, π_R, π_S are

$$[R(x), \pi_R(x')] = i\delta(x - x'), \quad [S(x), \pi_S(x')] = i\delta(x - x') \quad (4.4)$$

In terms of these fields, H_m may be written

$$H_m = \int dx [\pi_R \pi_S + \nabla R \cdot \nabla S + m^2 R S] \quad (4.5)$$

Some insight into the peculiar nature of H_m may be obtained by considering it to be the complete Hamiltonian, and by looking at the Heisenberg picture equations of motion

$$dR(x,t)/dt = i^{-1}[R, H] = \pi_S(x,t), \quad dS(x,t)/dt = \pi_R(x,t) \quad (4.6a,b)$$

$$d\pi_S(x,t)/dt = \nabla^2 R(x,t) - m^2 R(x,t), \quad d\pi_R(x,t)/dt = \nabla^2 S(x,t) - m^2 S(x,t) \quad (4.6c,d)$$

That is, $R(x,t)$ and $S(x,t)$ each obey the Klein-Gordon equation, but the time derivative of each field is the conjugate momentum of the other field!

Solving the Klein-Gordon equation, we have

$$R(x,t) = \int dk [R(x',t) \dot{G}(x-x',t) + \pi_S(x',t) G(x-x',t)] \quad (4.7)$$

where G is the retarded greens function (3.25). We see from Eq. (4.7) (and a similar expression for S) that

$$[R(x,t), R(x',t')] = 0, \quad [S(x,t), S(x',t')] = 0 \quad (4.8a,b)$$

i.e. *these commutators not only vanish for spacelike separations of (x,t) , (x',t') but for all separations.*

To examine the spectrum of H_m , it is convenient to introduce a modal representation for $R(x)$ and $S(x)$ in terms of Hermitian operators $q_1(k)$, $p_1(k)$:

$$R(x) = (2\pi)^{-3/2} \int dk (4\epsilon)^{-1/2} [q_1(k) - iq_2(k)] e^{ik \cdot x} + (q_1(k) + iq_2(k)) e^{-ik \cdot x} \quad (4.9a)$$

$$S(x) = (2\pi)^{-3/2} \int dk (4\epsilon)^{-1/2} i [p_1(k) - ip_2(k)] e^{ik \cdot x} - (p_1(k) + ip_2(k)) e^{-ik \cdot x} \quad (4.9b)$$

Eq. (4.9a) (or (4.9b)) is the most general expression one can write down for a real operator. Since $R(x,t)$ ($S(x,t)$) obeys the Klein-Gordon equation and equals $R(x)$ ($S(x)$) at $t=0$, $R(x,t)$ and $S(x,t)$ can be constructed by replacing $k \cdot x$ with $k \cdot x - \epsilon t$

in Eqs. (4.9). Then it follows from Eqs. (4.4), (4.6), (4.8) that the only nonvanishing commutators of q_i , p_j are

$$[q_1(k), p_1(k')] = i\delta(k - k') \quad (4.10)$$

Note that in particular, unlike the case for the usual scalar field, the positive frequency operator $q_1 - iq_2$ and the negative frequency operator $q_1 + iq_2$ commute with each other (similarly for $p_1 - ip_2$, $p_1 + ip_2$). We will use the representation (4.9) of R and S in terms of q_i , p_i as well as

$$\pi_R(x) = (2\pi)^{-3/2} \int dk (e/4)^{1/2} i [p_1(k) - ip_2(k)] e^{ik \cdot x} + (p_1(k) + ip_2(k)) e^{-ik \cdot x} \quad (4.11a)$$

$$\pi_S(x) = (2\pi)^{-3/2} \int dk (e/4)^{1/2} i [-q_1(k) - iq_2(k)] e^{ik \cdot x} + (q_1(k) + iq_2(k)) e^{-ik \cdot x} \quad (4.11b)$$

even when considering interacting fields.

Substituting Eqs. (4.9), (4.11) into Eq. (4.5) we obtain for H_m the following expression:

$$H_m = \int dk \epsilon(k) [q_1(k) p_2(k) - q_2(k) p_1(k)] \quad (4.12)$$

Thus, for the combined meson fields, *the spectrum of each mode is the spectrum of angular momentum*. Each mode can be regarded as analogous to a (zero kinetic energy) orbiting particle in the x-y plane in a z-directed magnetic field of strength $e(k)$. The "magnetic energy" spectrum $ne(k)$ (n can be a negative or positive integer or zero) is just what one expects from the combined modes of the positive energy and negative energy meson fields.

4b. Vacuum Behavior

In the model of section 3 we saw, due to reduction, that $d\bar{H}(t)/dt$ was infinite. Let us calculate the rate of energy increase due to reduction in the present model. The density matrix evolution equation corresponding to Eq. (4.1a) is

$$dD(t)/dt = -i[H, D(t)] - \frac{\lambda}{2} \int dx [R(x), [R(x), D(t)]] \quad (4.13)$$

Therefore

$$\begin{aligned} d\bar{H}(t)/dt &= d\text{Tr}\{\bar{H}D(t)\}/dt = -\frac{\lambda}{2} \text{Tr}\{\int d\mathbf{x}[\mathbf{R}(\mathbf{x}), [\mathbf{R}(\mathbf{x}), \bar{H}]]D(t)\} \\ &= -\frac{\lambda}{2} \text{Tr}\{\int d\mathbf{x}[\mathbf{R}(\mathbf{x}), \pi_S(\mathbf{x})]D(t)\} = 0 \end{aligned} \quad (4.14)$$

Thus the most obvious sign of distress of the previous model is not present here. But, what about the rate of increase of the higher moments of $\bar{H}$? For example,

$$d\bar{H}(t)^2/dt = \lambda \text{Tr}\{\int d\mathbf{x} \pi_S(\mathbf{x})^2 D(t)\} \quad (4.15)$$

is positive definite: is it infinite? We will now consider the vacuum evolving by itself, and show that a normalized "vacuum" state can be chosen for which $\bar{H}(t)^2$ and higher moments can have as small a rate of increase as one wishes.

When $\bar{H}$ is replaced by H_m in Eq. (4.13), and the modal decomposition of the previous section is employed, Eq. (4.13) becomes

$$\begin{aligned} dD(t)/dt &= -i\int d\mathbf{k} e(\mathbf{k})[q_1(\mathbf{k})p_2(\mathbf{k}) - q_2(\mathbf{k})p_1(\mathbf{k}) + q_1(-\mathbf{k})p_2(-\mathbf{k}) - q_2(-\mathbf{k})p_1(-\mathbf{k}), D(t)] \\ &\quad - (\lambda/2)\int d\mathbf{k} (2\epsilon)^{-1}([q_1(\mathbf{k}) + q_1(-\mathbf{k}), [q_1(\mathbf{k}) + q_1(-\mathbf{k}), D(t)]] + \\ &\quad [q_2(\mathbf{k}) - q_2(-\mathbf{k}), [q_2(\mathbf{k}) - q_2(-\mathbf{k}), D(t)]]) \end{aligned} \quad (4.16)$$

(Because the reduction part of the equation couples modes with $\mathbf{k}$ and $-\mathbf{k}$, in Eq. (4.16) we explicitly display the $-\mathbf{k}$ modes in the expression for H_m .) We now introduce "center of mass" variables Q_i , P_i and "relative" variables r_i , s_i :

$$Q_i(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [q_i(\mathbf{k}) + q_i(-\mathbf{k})], \quad P_i(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [p_i(\mathbf{k}) + p_i(-\mathbf{k})] \quad (4.17a,b)$$

$$r_i(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [q_i(\mathbf{k}) - q_i(-\mathbf{k})], \quad s_i(\mathbf{k}) \equiv \frac{1}{\sqrt{2}} [p_i(\mathbf{k}) - p_i(-\mathbf{k})] \quad (4.17c,d)$$

with the usual commutation relations so that Eq. (4.16) may be rewritten

$$dD(t)/dt = -i\int d\mathbf{k} e[\mathbf{Q}_1 P_2 - \mathbf{Q}_2 P_1 + r_1 s_2 - r_2 s_1, D(t)]$$

$$- (\lambda/2)\int d\mathbf{k} e^{-1}([Q_1, [Q_1, D(t)]] + [r_2, [r_2, D(t)]]) \quad (4.18)$$

For illustrative purposes, we will restrict our considerations to a particularly simple class of "vacuum" states³³. Going over to a discrete basis (as in section 3b),

and writing the density matrix as the product of density matrices $\tilde{D}_c(\mathbf{k}, t)$ and $\tilde{D}_r(\mathbf{k}, t)$ for each mode, we seek the solution of

$$d\tilde{D}_c/dt = -ie[\mathbf{Q}_1 P_2 - \mathbf{Q}_2 P_1, \tilde{D}_c] - (\lambda/2\epsilon)[Q_1, [Q_1, \tilde{D}_c]] \quad (4.19a)$$

$$d\tilde{D}_r/dt = -ie[r_1 s_2 - r_2 s_1, \tilde{D}_r] - (\lambda/2\epsilon)[r_2, [r_2, \tilde{D}_r]] \quad (4.19b)$$

(The same symbols are being employed for the discrete operators as for the continuous operators, e.g. $Q_1(\mathbf{k}) \rightarrow (\Delta k)^{-3/2} Q_1$ without, we hope, causing confusion.)

Eqs. (4.19) describe the "particle-in-a-magnetic-field" with reduction on one of the position coordinates. They are most easily solved in the "position" basis, with employment of polar coordinate operators θ, ϑ

$$Q_1 = Q \cos \theta, \quad Q_2 = Q \sin \theta, \quad r_1 = r \cos \vartheta, \quad r_2 = r \sin \vartheta \quad (4.20)$$

so that Eqs. (4.19) become

$$d\langle Q, \theta | \tilde{D}_c | Q', \theta' \rangle / dt = \left\{ -ie \left[\frac{1}{i} \frac{\partial}{\partial \theta} + \frac{1}{i} \frac{\partial}{\partial \theta'} \right] \right.$$

$$\left. - \frac{\lambda}{2\epsilon} [Q \cos \theta - Q' \cos \theta']^2 \right\} \langle Q, \theta | \tilde{D}_c | Q', \theta' \rangle \quad (4.21a)$$

$$d\langle r, \vartheta | \tilde{D}_r | r', \vartheta' \rangle / dt = \left\{ -ie \left[\frac{1}{i} \frac{\partial}{\partial \vartheta} + \frac{1}{i} \frac{\partial}{\partial \vartheta'} \right] \right.$$

$$\left. - \frac{\lambda}{2\epsilon} [r \sin \vartheta - r' \sin \vartheta']^2 \right\} \langle r, \vartheta | \tilde{D}_r | r', \vartheta' \rangle \quad (4.21a)$$

Before solving Eqs. (4.21) in general, we want to digress a bit to focus on a particular class of solutions, described by the (improper, because it is unnormalized) pure state density matrix

$$\langle Q, \theta | \tilde{D}_c | Q', \theta' \rangle = \delta(\theta - \theta_0 - \epsilon t) \delta(Q - Q_0) \delta(\theta' - \theta_0 - \epsilon t) \delta(Q' - Q_0) \quad (4.22a)$$

$$\langle r, \vartheta | \tilde{D}_r | r', \vartheta' \rangle = \delta(\vartheta - \vartheta_0 - \epsilon t) \delta(r - r_0) \delta(\vartheta' - \vartheta_0 - \epsilon t) \delta(r' - r_0) \quad (4.22b)$$

Eq. (4.22a), for example, corresponds to the wavefunction of a perfectly localized particle at radius Q_0 rotating in the $Q_1 - Q_2$ plane with angular velocity ϵ . We will later see that the reduction proceeds toward an ensemble of normalized states, each approximating (4.22).

The pure states $|r, \vartheta\rangle, |Q, \theta\rangle, |r_0(\mathbf{k}), \vartheta_0(\mathbf{k})\rangle, |r_0(\mathbf{k}), \vartheta_0(\mathbf{k})\rangle$, characterized by the validity of Eqs. (4.22) for all modes, are eigenstates of $\mathbf{R}(\mathbf{x})$ (as well as $\pi_S(\mathbf{x})$) at each instant of

time. We can express $R(\mathbf{x})$ in terms of the operators $Q_1(\mathbf{k})$, $r_2(\mathbf{k})$, using Eqs. (4.9a), (4.17) and (4.20):

$$R(\mathbf{x}) = (2\pi)^{-3/2} \int d\mathbf{k} (2/\epsilon)^{1/2} [Q_1(\mathbf{k}) \cos \mathbf{k} \cdot \mathbf{x} + r_2(\mathbf{k}) \sin \mathbf{k} \cdot \mathbf{x}] \quad (4.23)$$

Therefore the eigenvalue $r(\mathbf{x}, t)$ of $R(\mathbf{x})$ for such a state is given by

$$r(\mathbf{x}, t) = (2\pi)^{-3/2} \int d\mathbf{k} (2/\epsilon)^{1/2} [Q_0(\mathbf{k}) \cos(\theta_0 + \epsilon t) \cos \mathbf{k} \cdot \mathbf{x} + r_2(\mathbf{k}) \sin(\theta_0 + \epsilon t) \sin \mathbf{k} \cdot \mathbf{x}] \quad (4.24)$$

We wish to point out that $r(\mathbf{x}, t)$ is a *sample random field obeying the Klein-Gordon equation*, if $Q_0, \theta_0, r_0, \dot{\theta}_0$ are random functions of $\mathbf{k}$. But one *would* expect them to be random functions of $\mathbf{k}$ if the state $|\psi, Q_0, \theta_0, r_0, \dot{\theta}_0\rangle$ (more precisely, a normalized close approximation to it) is the result of reduction dynamics acting on an arbitrary initial vacuum statevector, since each mode reduces independently and randomly. [We note that $|\psi, Q_0, \theta_0, r_0, \dot{\theta}_0\rangle$ satisfies the dynamical equation (4.1a) (with $H_1 = H_2 = 0$) for any fluctuation $w(\mathbf{x}, t)$. The reduction terms just produce an inessential time dependent c -number that affects the norm of the statevector, but not the dynamics under the Hermitian Hamiltonian H_m which the statevector obeys - that is, $|\psi, Q_0, \theta_0, r_0, \dot{\theta}_0\rangle$ is a completely reduced statevector.]

Thus, totally unexpectedly, without putting in any white noise-dependent reduction term, there appears a random function of time arising in a natural way from the "vacuum dynamics" alone. Here we have a class of time-dependent "vacuum" states which are eigenstates of $R(\mathbf{x})$ with random field eigenvalues. Then we find, when we couple $R(\mathbf{x})$ to the white noise to obtain reduction, that the reduction strives to produce this same class of states. This may lead one to hope that one could dispense with the external white noise entirely, obtaining the fluctuations entirely from within the theory. To be sure, $r(\mathbf{x}, t)$ satisfies the Klein-Gordon equation and $w(\mathbf{x}, t)$ does not. Moreover, the probability distribution of $Q_0, \theta_0, r_0, \dot{\theta}_0$ determines the statistical behavior of $r(\mathbf{x}, t)$, and no apparent mechanism for achieving the statistical behavior appropriate for the probability rule has yet appeared. Nonetheless, we believe this is an exciting possibility that merits further investigation.

Returning now to Eqs. (4.21) we will illustrate "vacuum" dynamics with a particular choice of an initial state of the "vacuum" with zero energy (no angular dependence, the zero "angular momentum" state for each mode):

$$\langle Q, \theta | \tilde{D}_c(0) | Q, \theta \rangle = f(Q) f^*(Q) \quad (4.25)$$

and similarly for $\tilde{D}_r(t)$. The solution of Eq. (4.21a) with initial condition (4.25) is

$$\langle Q, \theta | \tilde{D}_c(t) | Q, \theta \rangle = f(Q) f^*(Q) e^{-(\lambda/4\epsilon)(Q - Q')^2}.$$

$$-e^{(\lambda/8\epsilon^2)(Q^2/2 \sin^2(\theta - \epsilon t) - \sin^2 \theta) + Q^2/2 \sin^2(\theta - \epsilon t) - \sin^2 \theta} [2QQ'(\sin(\theta + \theta' - 2\epsilon t)) \sin(\theta + \theta')] \quad (4.26)$$

($Q = iQ \cos \theta + jQ \sin \theta$) and a similar result holds for $\tilde{D}_r(t)$.

The major feature of Eq. (4.26) is the exponential decay of the off-diagonal elements of $\tilde{D}_c(t)$ toward a mixture of eigenstates of Q : this shows the reduction striving toward eigenstates of $R(\mathbf{x})$ that has been claimed. However we note that, for example, if $f(Q(\mathbf{k}))$ is only nonvanishing for $Q(\mathbf{k}) < \rho(\mathbf{k})$, where $\rho(\mathbf{k})$ is a small radius, the exponential decay is no faster than $\exp(-\lambda \rho(\mathbf{k})^2/4\epsilon(\mathbf{k}))$. This suggests that we can build up a normalized vacuum state by making $\rho(\mathbf{k})$ sufficiently small for each $\mathbf{k}$ so that the state departs from its original form as slowly as we wish.

In addition we will require $\rho(\mathbf{k} \rightarrow \infty) \rightarrow 0$ sufficiently rapidly so that the state $|\nu\rangle$ (the direct product of normalized mode states) satisfies $\|R(\mathbf{x})|\nu\rangle\| \ll 1$, $\|r_S(\mathbf{x})|\nu\rangle\| \ll 1$:

$$R(\mathbf{x}) |\nu\rangle \approx 0, \quad \pi_S(\mathbf{x}) |\nu\rangle \approx 0 \quad (4.27a, b)$$

We shall call Eqs. (4.27a, b) the "gauge" conditions, and any state satisfying them will be regarded as a suitable "vacuum" state. Although any single state $|\nu\rangle$ is not Lorentz invariant³³ (but the ensemble of suitable $|\nu\rangle$'s is), the conditions (4.27) are Lorentz invariant ($\|R(\mathbf{x})|\nu\rangle\|$, $\|r_S(\mathbf{x})|\nu\rangle\|$ are invariant under unitary Lorentz transformations), and they are all that we shall need when we consider adding fermions.

An important question is, can the "gauge" conditions be maintained when we add the fermions to the theory? Without reduction the answer is yes. We will see in this case that we have a theory of free fermions, and $|\nu\rangle$ can be maintained as the state which the dressed free fermions creation operators act upon. When there is reduction, the fermions interact with the vacuum. In the extreme nonrelativistic limit we shall find, as in section 3d, that the interaction still does not appreciably disturb $|\nu\rangle$, so that the "gauge" conditions can still be maintained to any desired degree of accuracy. However, when the fermions move this is an unanswered question of great importance, to which we will return in section 4e.

Finally, we wish to address the question of energy production for a "vacuum" state due to the reduction. For example, starting with a zero energy-angular momentum state of a mode, the reduction dynamics breaks up the original wavefunction, which is independent of angle, into wavepackets that are more and more localized in angle. Such increasingly localized states are built up of angular momentum of increasing magnitude, so we expect the energy to monotonically increase due to reduction. To estimate this increase we consider an initial state $f(Q) = \delta^{1/2}(Q - Q_0) / \sqrt{2\pi}$ (i.e., a state of

zero angular momentum that is well localized at $Q=Q_0$). Disregarding the fluctuating part of Eq. (4.26), the density matrix is

$$\langle Q, \theta | \tilde{D}_c(t) | Q', \theta' \rangle \approx (2\pi)^{-1} \delta^{1/2}(Q-Q_0) \delta^{1/2}(Q'-Q_0) e^{i(\lambda/4e) Q_0^2 \sin^2 \frac{1}{2}(\theta-\theta')} \quad (4.28)$$

The first two moments of the energy associated with each mode are

$$\bar{H} \equiv \text{Tr} \left\{ e^{-\frac{1}{\partial\theta}} \langle Q, \theta | \tilde{D}_c(t) | Q', \theta' \rangle \right\} = 0 \quad (4.29a)$$

$$\bar{H}^2 \equiv \text{Tr} \left\{ e^{-2\frac{1}{\partial\theta}} \langle Q, \theta | \tilde{D}_c(t) | Q', \theta' \rangle \right\} = \lambda_1 Q_0^2 e/2 \quad (4.29a)$$

Eq. (4.29a) is just the statement for a single mode of the more general result (4.14), that the average energy remains constant. But Eq. (4.29b) answers the question about energy increase raised in connection with Eq. (4.15). We see that there is an energy increase per mode, as expressed in the higher moments of H , but that it can be made as small as we wish by choosing Q_0 to be sufficiently small. In particular, the "gauge" selected "vacuum" state $|v\rangle$ can be made to have an arbitrarily small energy increase.

4c. Dressed Fermions: Extreme Nonrelativistic Limit

Before we can consider reduction of fermion states, we must understand the nature of those states without reduction. In this section we add to the meson Hamiltonian the interaction Hamiltonian H_i (appropriate for the extreme nonrelativistic limit), but the fermion Hamiltonian H_f is still set equal to zero. In the next two sections we will successively consider the field theory with a nonrelativistic H_f , and finally the relativistic H_f .

A quick way of seeing how the fermion gets dressed is to write the Hamiltonian as

$$\begin{aligned} H &= \int d\mathbf{x} \{ \pi_R(\mathbf{x}) \pi_S(\mathbf{x}) + S(\mathbf{x}) [-\nabla^2 R(\mathbf{x}) + m^2 R(\mathbf{x}) + g \xi^\dagger(\mathbf{x}) \xi(\mathbf{x})] \} \\ &= \int d\mathbf{x} \{ \pi_R(\mathbf{x}) \pi_S(\mathbf{x}) + S(\mathbf{x}) [-\nabla^2 + m^2] (R(\mathbf{x}) + g \int d\mathbf{x}' \xi^\dagger(\mathbf{x}') \xi(\mathbf{x}') y(\mathbf{x}-\mathbf{x}')) \} \end{aligned} \quad (4.30)$$

We observe that those fermion states which are (simultaneous) eigenstates of the (commuting) operators $\pi_S(\mathbf{x})$ and $R(\mathbf{x}) + g \int d\mathbf{x}' \xi^\dagger(\mathbf{x}') \xi(\mathbf{x}') y(\mathbf{x}-\mathbf{x}')$, with eigenvalues zero, are eigenstates of H with energy 0. In fact, it is easy to see that such states are

$$| \mathbf{x}_1, \dots, \mathbf{x}_N \rangle \equiv \prod_{n=1}^N e^{ig \int d\mathbf{x} \pi_R(\mathbf{x}) y(\mathbf{x}_n - \mathbf{x})} \xi^\dagger(\mathbf{x}) | v \rangle \quad (4.31)$$

Since π_S commutes with π_R and $\xi^\dagger$, and $\pi_S | v \rangle = 0$, we have

$$\pi_S(\mathbf{x}) | \mathbf{x}_1, \dots, \mathbf{x}_N \rangle = 0 \quad (4.32)$$

The states (4.31) are also eigenstates of $R(\mathbf{x})$:

$$R(\mathbf{x}) | \mathbf{x}_1, \dots, \mathbf{x}_N \rangle = -g \sum_{n=1}^N y(\mathbf{x}_n - \mathbf{x}) | \mathbf{x}_1, \dots, \mathbf{x}_N \rangle \quad (4.33)$$

Therefore, if we have a superposition of the states (4.31), each corresponding to a particular spatial configuration of fermions, it is a superposition of different eigenstates of $R(\mathbf{x})$. However, each state carries its signature, an eigenvalue "handle" that is the sum of Yukawa-potentials attached to that configuration of fermions. Therefore, when we couple $R(\mathbf{x})$ to the white noise field, it will strive to reduce the superposition to one such configuration of fermions (see section 4f).

In the remainder of this section, we will show that the states (4.31) are the natural dressed fermion states to consider. We follow the procedure of section 3c, defining the bare N -fermion state by Eq. (3.19) (with $|0\rangle$ there replaced by $|v\rangle$). Again, denoting $\tilde{\xi}^\dagger(\mathbf{x}, t) \equiv (\exp(-iHt)) \xi^\dagger(\mathbf{x}) (\exp(iHt))$ as the operator describing the fermion being dressed, we have, as in Eq. (3.20), a solution of the Schrödinger equation

$$| \mathbf{v}, t \rangle = \tilde{\xi}^\dagger(\mathbf{x}_1, t) \dots \tilde{\xi}^\dagger(\mathbf{x}_N, t) | v \rangle \quad (4.34)$$

(We have used $\exp(-iHt) | v \rangle = 0$, which follows from the construction of $|v\rangle$ of the previous section, and $H_f | v \rangle = 0$).

We now calculate $\tilde{\xi}^\dagger(\mathbf{x}, t)$. Using Eq. (3.22) with the Hamiltonian (4.30) we obtain, as in Eqs. (3.23),

$$d\tilde{S}/dt = -\tilde{\pi}_R, \quad \square + m^2 \tilde{S} = 0, \quad d\tilde{\xi}^\dagger/dt = -ig \tilde{\xi}^\dagger \tilde{S}, \quad (4.35a, b, c)$$

Since $\tilde{S}$ is a free field (and that is what makes this a solvable model of free fermions) we can write

$$\tilde{S}(\mathbf{x},t) = [d\mathbf{x}' [S(\mathbf{x}')\tilde{G}(\mathbf{x}-\mathbf{x}',t)-\pi_R(\mathbf{x}')\tilde{G}(\mathbf{x}-\mathbf{x}',t)]] \quad (4.36)$$

where G is the retarded greens function (3.25). The solution of Eq. (4.35c) is

$$\tilde{\xi}^{\dagger}(\mathbf{x},t) = e^{-ig\int_0^t dt} \tilde{S}(\mathbf{x},t) \xi^{\dagger}(\mathbf{x}) \quad (4.37)$$

since $\tilde{S}(\mathbf{x},t)$ commutes with itself at all times.

We are interested in the completely dressed fermion creation operator $\tilde{\xi}^{\dagger}(\mathbf{x},\infty) \equiv \tilde{\xi}^{\dagger}(\mathbf{x})$. As in section 3c, we perform the time integral in Eq. (4.37) over the greens functions using Eqs. (3.28), obtaining the results (3.29), and thereby

$$\tilde{\xi}^{\dagger}(\mathbf{x}) = e^{ig\int d\mathbf{x}' \pi_R(\mathbf{x}')y(\mathbf{x}-\mathbf{x}')\xi^{\dagger}(\mathbf{x})} \quad (4.38)$$

This result was used in constructing the states (4.31). It shows that the dressing of a fermion at $\mathbf{x}$ consists of multiplying it by a translation operator which, when acting on the "vacuum" state $|v\rangle$ which is an eigenstate of $R(y)$ with eigenvalue 0, translates it to an eigenstate of $R(y)$ with eigenvalue $-gy(\mathbf{x}-y)$, as indicated in Eq. (4.33).

The state (4.31) constructed from the product of dressed operators (4.38) is an eigenstate of the Hamiltonian with eigenvalue zero: this follows from

$$[H, \tilde{\xi}^{\dagger}(\mathbf{x})] = 0 \quad (4.39)$$

which may readily be directly calculated. Thus the state (4.31), or any superposition of such states, which describes dressed fermions at rest, is a solution of Schrödinger's evolution equation, although it has no time dependence.

In the next section we will add fermion kinetic energy to the Hamiltonian and construct time dependent solutions of Schrödinger's equation by the same method. In fact, this is just a method for creating in-states, and $\tilde{\xi}^{\dagger}(\mathbf{x})$ might be called an in-state creation operator. The unitary operator that creates in-states, which is

$$U(0,-\infty) \equiv \lim_{t_0 \rightarrow -\infty} e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0} = e^{ig\int d\mathbf{x}' \pi_R(\mathbf{x}')y(\mathbf{x}-\mathbf{x}')\xi^{\dagger}(\mathbf{x})\xi(\mathbf{x}')} \quad (4.40)$$

(in this case $H_0 = H_m$, $H = H_m + H_1$) satisfies

$$U\xi^{\dagger}(\mathbf{x})U^{\dagger} = \tilde{\xi}^{\dagger}(\mathbf{x}) \quad (4.41)$$

so that the state (4.31) consisting of N dressed fermions is just the in-state:

$$|x_1, \dots, x_N\rangle = U\xi^{\dagger}(x_1) \dots \xi^{\dagger}(x_N)|v\rangle \quad (4.42)$$

4d. Dressed Fermions: Nonrelativistic Limit

We will now see some new features which arise when we add

$$H_f \equiv \int d\mathbf{x} \xi^{\dagger}(\mathbf{x})(2M)^{-1}(-\nabla^2)\xi(\mathbf{x}) \quad (4.43)$$

to the Hamiltonian (4.40). It is more complicated because Eq. (4.35c) must be replaced by

$$\partial \tilde{\xi}^{\dagger}(\mathbf{x},t)/\partial t = -i[-\frac{1}{2M}\nabla^2 + g e^{-\delta t} \tilde{S}(\mathbf{x},t)]\tilde{\xi}^{\dagger}(\mathbf{x},t) \quad (4.44)$$

(The convergence factor $\exp(-\delta t)$ has been inserted in anticipation of taking the $t \rightarrow \infty$ limit, as explained following Eq. (3.28).) The solution to Eq. (4.44) is

$$\tilde{\xi}^{\dagger}(\mathbf{x},t) = e^{-it(2M)^{-1}\nabla^2} \tilde{\xi}_t^{\dagger}(\mathbf{x}) \quad (4.45a)$$

$$\tilde{\xi}_t^{\dagger}(\mathbf{x}) \equiv T\{\exp[-ig\int_0^t dt' e^{-\delta t'} e^{-it'(2M)^{-1}\nabla^2} \tilde{S}(\mathbf{x},t')] e^{it'(2M)^{-1}\nabla^2}\} \xi^{\dagger}(\mathbf{x}) \quad (4.45b)$$

where $\tilde{S}(\mathbf{x},t)$ is given in terms of $S(\mathbf{x})$, $\pi_R(\mathbf{x})$ and the greens function G by Eq. (4.36). The time ordered product in Eq. (4.42) is necessary because ∇^2 does not commute with G .

We define the dressed fermion creation operators as

$$\tilde{\xi}^{\dagger}(\mathbf{x}) \equiv \lim_{t \rightarrow \infty} e^{-it(2M)^{-1}\nabla^2} \tilde{\xi}_t^{\dagger}(\mathbf{x},t) = \tilde{\xi}_{\infty}^{\dagger}(\mathbf{x}) \quad (4.46)$$

These are, in fact, just in-state creation operators as in the previous section, satisfying Eq. (4.41) with

$$U(0,-\infty) = T\{\exp[-ig\int_{-\infty}^0 dt' e^{\delta t'} \int d\mathbf{x}' \tilde{S}(\mathbf{x},t') [e^{-it'(2M)^{-1}\nabla^2} \xi^{\dagger}(\mathbf{x})] [e^{it'(2M)^{-1}\nabla^2} \xi(\mathbf{x}')]]\} \quad (4.47)$$

We can use these operators to construct solutions of the Schrödinger equation for free dressed fermions, using the property

$$[H, \tilde{\xi}_1^\dagger(\mathbf{x})] = -(2M)^{-1} \nabla^2 \tilde{\xi}_1^\dagger(\mathbf{x}) \quad (4.48)$$

(Eq. (4.47) follows from equating $d\tilde{\xi}_1^\dagger(\mathbf{x},t)/dt = -i[H, \tilde{\xi}_1^\dagger(\mathbf{x},t)]$ to Eq. (4.44), multiplying by $\exp(-it(2M)^{-1}\nabla^2)$, and taking the $t \rightarrow \infty$ limit.) That

$$|v, \Delta_N\rangle = \int d\mathbf{x}_N \cdot d\mathbf{x}_1 [e^{it(2M)^{-1}\nabla_N^2} f(\mathbf{x}_N)] \cdot [e^{it(2M)^{-1}\nabla_1^2} f(\mathbf{x}_1)] \tilde{\xi}_1^\dagger(\mathbf{x}_N) \cdot \tilde{\xi}_1^\dagger(\mathbf{x}_1) |v\rangle \quad (4.49)$$

satisfies $i d|v, \Delta_N\rangle/dt = H|v, \Delta_N\rangle$ follows directly from differentiation of Eq. (4.49) and use of Eq. (4.48), as well as $H|v\rangle = 0$.

What makes Eq. (4.49) the description of a bunch of noninteracting fermions is the fact that the time-ordered product in

$$\begin{aligned} \tilde{\xi}_1^\dagger(\mathbf{x}_N) = & T \left[\exp \left[-ig \int_0^\infty dt' e^{-\delta t'} e^{-it'(2M)^{-1}\nabla_N^2} \right. \right. \\ & \left. \left. \cdot \int d\mathbf{x}' [S(\mathbf{x}') G(\mathbf{x}_N - \mathbf{x}', t') - \pi R(\mathbf{x}') G(\mathbf{x}_N - \mathbf{x}', t')] \right] e^{it'(2M)^{-1}\nabla_N^2} \right] \tilde{\xi}_1^\dagger(\mathbf{x}_N) \end{aligned} \quad (4.50)$$

that is associated to a dressed fermion at $\mathbf{x}_N$ commutes with those associated to fermions at other positions and does not depend upon fermion creation and annihilation operators, as opposed to the case in a field theory where fermions interact. Due to the form of (4.50), Eq. (4.49) describes the evolution of N fermions, each independently travelling along accompanied by its "vacuum" excitations.

This "vacuum" excitation spectrum, and the fermion momentum spectrum that is associated with a one-particle in-state $|v, \Delta\rangle$ of energy $E_p = \mathbf{p}^2/2M$ and momentum $\mathbf{p}$ (e.g., set $f(\mathbf{x}_1) = \exp i\mathbf{p} \cdot \mathbf{x}_1$ in Eq. (4.49) for $|v, \Delta\rangle_1$) deserves further comment. The S -matrix $U(\infty, -\infty) = U^\dagger(0, \infty)U(0, -\infty)$ can be found from Eq. (4.47) and the out-state unitary operator $U^\dagger(0, \infty)$ (Eq. (4.47)) with $g \int_{-\infty}^0 dt'$ replaced by $-g \int_0^\infty dt'$ or, more simply, employing

$$U(\infty, -\infty) = T \left\{ \exp \left[-ig \int d^4x e^{-\delta t} [\tilde{S}(\mathbf{x}, t) [e^{-it(2M)^{-1}\nabla^2} \tilde{\xi}_1^\dagger(\mathbf{x})] [e^{it(2M)^{-1}\nabla^2} \tilde{\xi}_1(\mathbf{x})]] \right] \right\} \quad (4.51)$$

its use may be illustrated by the first-order expression for the amplitude to go from the state $|p\rangle |v\rangle$ to $|p'\rangle |v'\rangle$, where $\mathbf{p}$ is the fermion momentum ($|p\rangle \equiv (2\pi)^{-3/2} \int d\mathbf{x} e^{i\mathbf{p} \cdot \mathbf{x}} \tilde{\xi}_1^\dagger(\mathbf{x}) |0\rangle$) and v_i, v_f refer to the initial and final states of the vacuum:

$$\langle p', v_f | U(\infty, -\infty) | p, v \rangle = \delta(\mathbf{p} - \mathbf{p}') \delta(v_f, v_i) - 2\pi i \delta(\mathbf{k} \delta(\mathbf{p} - \mathbf{p}') \delta(\mathbf{g}/2) e^{i\mathbf{k} \cdot \mathbf{x}} \cdot$$

$$\cdot \{ \delta(E_p - E_{p'} - E_k) \langle v_f | S(\mathbf{x}) - i e_k^{-1} \pi_R(\mathbf{x}) | v_i \rangle +$$

$$+ \delta(E_p - E_{p'} + E_k) \langle v_f | S(\mathbf{x}) + i e_k^{-1} \pi_R(\mathbf{x}) | v_i \rangle \} \quad (4.52)$$

This shows that at the basic Feynman diagram vertex, a fermion of energy and momentum $(E_p, \mathbf{p})$ either emits or absorbs a meson of positive or negative energy $\pm E_k$, thereby becoming a fermion characterized by $(E_{p'}, \mathbf{p}')$. It happens that the rate calculated from Eq. (4.52) vanishes because such a real process violates energy-momentum conservation. (A fermion at rest ($\mathbf{p}=0$), that decays to a moving fermion and a negative energy meson requires a fermion final energy $E_{p'} = 2M$, which clearly is out of the realm of the nonrelativistic approximation. When the relativistic expression for the fermion energy is used, there is no solution provided $m/2M < 1$, which we assume.) However, in higher orders there are allowed processes involving two or more mesons, e.g. a fermion at rest suddenly moves forward emitting two negative energy mesons in the backward direction. However, these cannot be regarded as *real* mesons because, as we see from Eq. (4.49), they do not collide with other dressed fermions when these are present. So perhaps we should say that a fermion travels along accompanied by "real" vacuum excitations, instead of possessing just the usual "virtual" excitations of a standard quantum field theory which has only positive energy mesons.

It is interesting that we have here, without reduction, a situation similar to that which occurs when there is reduction, a particle propagating through an infinite medium (the "vacuum") with which it continuously interacts. Whether a wavepacket state $|v, \Delta\rangle_1$ describes a particle in some kind of stasis with its "vacuum" excitation environment, or perhaps whether, as in the nonrelativistic CSL reduction theory, the particle's energy continues to grow as negative energy mesons are produced, or whether something else happens, is an important question for future analysis.

We close this section by quoting without proof that $|v, \Delta\rangle_1$, to first order in g and with some approximations appropriate to a nonrelativistic theory (i.e. wavefunction momenta $\ll M$, and $m \ll M$), is given by

$$\begin{aligned} |v, \Delta\rangle_1 = & \int d\mathbf{x} [e^{it(2M)^{-1}\nabla^2} f(\mathbf{x}, t)] \cdot \\ & \cdot e^{ig \int d\mathbf{x}' [-S(\mathbf{x}') 2M \sin(M|\mathbf{x} - \mathbf{x}'|) + \pi R(\mathbf{x}') [1 - \cos(M|\mathbf{x} - \mathbf{x}'|)]]} y(\mathbf{x} - \mathbf{x}') \tilde{\xi}_1^\dagger(\mathbf{x}) \end{aligned} \quad (4.53)$$

In the $M \rightarrow \infty$ limit, where the sinusoids oscillate so rapidly that their net contribution vanishes, this becomes equivalent to (4.38). The major effect of finite M , to first order in g , is to alter the Yukawa-potential in the neighborhood of the origin, over a region of the size of the particle Compton wavelength, so that the effective potential is not infinite at the origin.

4e. Dressed Relativistic Fermions

We wish to construct dressed relativistic fermion creation operators by the method of section 3c that we have been employing. For this purpose, we need a fermion vacuum state, for the bare fermion field creation operators to act upon, that contains no bare fermions. For this reason, we find it advantageous to work with Dirac's original picture of positive and negative energy fermion creation operators acting on a no-fermion vacuum $|0\rangle_f$. Later on we will discuss filling up the negative energy states with dressed negative energy fermions (nefs) to create the physical fermion "vacuum."

The operator we will utilize to create a bare pef (positive energy fermion) or a bare nef at a point $\mathbf{x}$ is

$$\psi^{\dagger}(\mathbf{x}) = (2\pi)^{-3/2} [d(\mathbf{p})(M/E_p)^{1/2} \sum_{\mathbf{r}} [b_{\mathbf{r}}^{\dagger}(\mathbf{p})u_{\mathbf{r}}^*(\mathbf{p}) + c_{\mathbf{r}}^{\dagger}(\mathbf{p})v_{\mathbf{r}}^*(-\mathbf{p})] e^{-i\mathbf{p}\cdot\mathbf{x}} \quad (4.54)$$

($E_p \equiv \sqrt{p^2 + M^2}$). In Eq. (4.54), $b_{\mathbf{r}}^{\dagger}(\mathbf{p})$ and $c_{\mathbf{r}}^{\dagger}(\mathbf{p})$ create bare pefs and nef's respectively of momentum $\mathbf{p}$ and $u_{\mathbf{r}}(\mathbf{p})$ and $v_{\mathbf{r}}(\mathbf{p})$ are the usual positive and negative energy solutions of the Dirac equation with momentum $\mathbf{p}$.

The equation for $\tilde{\psi}^{\dagger}(\mathbf{x}, t)$, that replaces Eq. (4.44) for $\tilde{\xi}^{\dagger}(\mathbf{x}, t)$ of the nonrelativistic case, is

$$\begin{aligned} \partial \tilde{\psi}^{\dagger}(\mathbf{x}, t) / \partial t &= -i[\tilde{\psi}^{\dagger}(\mathbf{x}, t) \gamma^0 (-i\gamma \cdot \nabla + M) + g \tilde{\psi}^{\dagger}(\mathbf{x}, t) \gamma^0 \tilde{S}(\mathbf{x}, t)]_s \\ &= -i_s [(-i\gamma \cdot \nabla + M) \gamma^0 + g \tilde{S}(\mathbf{x}, t) \gamma^0] \tilde{\psi}^{\dagger}(\mathbf{x}, t) \end{aligned} \quad (4.55)$$

(In the last step of Eq. (4.55) we have employed the representation $\gamma^0 = (1, 1, -1, -1)$ on the diagonal and zero elsewhere, and $\gamma^T = -\gamma$, in order to convert $\tilde{\psi}^{\dagger}$ from a row matrix to a column matrix, which is more convenient.) Solution of this equation and the construction of the dressed fermion creation operator $\tilde{\psi}^{\dagger}(\mathbf{x})$ then follows the lines of the previous section, with the result parallel to that in Eq. (4.50):

$$\tilde{\psi}^{\dagger}(\mathbf{x}) = T \{ \exp[-ig \int_0^t dt' e^{it'(-i\gamma \cdot \nabla + M)} \gamma^0 \cdot \gamma^0 \int d\mathbf{x}' [S(\mathbf{x}') G(\mathbf{x} - \mathbf{x}', t) - \pi_R(\mathbf{x}') G(\mathbf{x} - \mathbf{x}', t')] e^{-it'(-i\gamma \cdot \nabla + M)} \gamma^0] \} \psi^{\dagger}(\mathbf{x}) \quad (4.56)$$

As was done for Eq. (4.48), it can be shown that

$$[H, \tilde{\psi}^{\dagger}(\mathbf{x})] = \gamma^0 (-i\gamma \cdot \nabla + M) \tilde{\psi}^{\dagger}(\mathbf{x}) \quad (4.57)$$

Using Eq. (4.57) we can construct energy eigenstates (in-states) of the Hamiltonian, such as

$$|p, \pm E_p, r\rangle \equiv (M/E_p)^{1/2} w_{\pm}(\mathbf{p}) (2\pi)^{-3/2} \int d\mathbf{x} e^{i\mathbf{p}\cdot\mathbf{x}} \tilde{\psi}^{\dagger}(\mathbf{x}) |0\rangle_f \quad (4.58)$$

where if $w_1(\mathbf{p}) \equiv u_1(\mathbf{p})$ or $w_2(\mathbf{p}) \equiv u_2(\mathbf{p})$ is used, then these are pef states, while if $w_3(\mathbf{p}) \equiv v_1(-\mathbf{p})$ or $w_4(\mathbf{p}) \equiv v_2(-\mathbf{p})$ is used these are nef states. We note from Eq. (4.58) that e.g. the dressed pef state of momentum $\mathbf{p}$ will contain not only a superposition of bare pefs of momenta around $\mathbf{p}$, but also will contain bare nef's in the superposition, because of the $\mathbf{x}$ -dependence of the exponential factor in $\tilde{\psi}^{\dagger}(\mathbf{x})$ (Eq. (4.56)).

Likewise, solutions of the time-dependent Schrödinger equation can be constructed for N dressed fermions:

$$|\Psi, t\rangle_N = \int d\mathbf{x}_N \cdot d\mathbf{x}_1 f_N(\mathbf{x}_N) \cdot f_1(\mathbf{x}_1) \tilde{\psi}^{\dagger}(\mathbf{x}_N) \cdot \tilde{\psi}^{\dagger}(\mathbf{x}_1) |v\rangle_0 \quad (4.59a)$$

$$f_n = (2\pi)^{-3/2} [d(\mathbf{p}) g(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} \pm i E_n(M/E_p)^{1/2} w_{\pm}(\mathbf{p})] \quad (4.59b)$$

An interesting new feature of the relativistic situation is that the sign and effective magnitude of the g -charge depends upon the kind of fermion and its speed. To see this, let us make the approximation of neglecting $-i\gamma \cdot \nabla$ compared to M in the exponent of Eq. (4.56), obtaining

$$\tilde{\psi}^{\dagger}(\mathbf{x}) \approx e^{ig\gamma^0 \int d\mathbf{x}' \pi_R(\mathbf{x}') y(\mathbf{x} - \mathbf{x}') \psi^{\dagger}(\mathbf{x})} \quad (4.60)$$

Using this to construct a dressed fermion at $\mathbf{x}$, we find

$$R(y) \tilde{\psi}^{\dagger}(\mathbf{x}) |v\rangle_0 \approx -g y (\mathbf{x} - \mathbf{y}) \gamma^0_{ss} \psi^{\dagger}(\mathbf{x}) |v\rangle_0 \quad (4.61)$$

We see that when $s=1$ or 2 that the state in (4.61) is an eigenstate of $R(y)$ with positive g -charge, but when $s=3$ or 4 , the g -charge is negative. Suppose, for example, we make a wavepacket for a dressed net that is fairly localized about momentum p_0 and position x_0 . If $p_0 \ll M$ so that $\tilde{\psi}_3^+$ and/or $\tilde{\psi}_4^+$ are much larger than the other two components of $\tilde{\psi}^+$, then the wavepacket will be an approximate eigenstate of $R(y)$ with eigenvalue $+gy(x_0 \cdot y)$. However, as p_0 increases, the upper and lower components of $\tilde{\psi}^+$ approach comparable size, and the effective magnitude of g will decrease.

Since the reduction rate is proportional to g^2 (to lowest order), this means that a superposition of states of a particle, all of which have approximately the same *large* momentum, will reduce more slowly than it does in the rest frame of the particle. This might have been anticipated as a reflection of the time dilation effect. What might not have been anticipated is that a superposition of two states of a particle, where both states describe the particle as being in essentially the same place, but one state has a larger momentum than the other, will also reduce because the effective strength of the Yukawa-potential is different for the two states. (Of course, in this free-field theory, such particle states will soon spatially separate and reduce accordingly, but the phenomena could be expected to persist in a theory which allowed bound states.) Both effects merit further study.

Finally we should say something about the physical vacuum, constructed by filling all the negative energy states with dressed nets. The way this works may be seen by taking the product of these states, constructed as in Eq. (4.58), and using for $\tilde{\psi}^+(x)$ the expression (4.60) to first order in g . One obtains $\prod_{r=1,2} \prod_p c_r^\dagger(p) |v\rangle |0\rangle_f$ plus the same expression with one $c_r^\dagger(p)$ replaced by

$$\sum_{s=1,2} ig(M/E_p)^{1/2} [dk(M/E_k)^{1/2} v_f(-p) \gamma^0 u_s^*(k)] \cdot \int dx dx' \pi_R(x') \gamma(x-x') e^{ip \cdot x - ik \cdot x} b_s^\dagger(k) \quad (4.62)$$

So, although the fermion "vacuum" is mostly made up of a complete set of bare nets (all holes filled, no bare antifermions) there is an amplitude for the absence of a bare net and the presence of a bare pef (a bare fermion-antifermion pair) together with a disturbance of the meson "vacuum" (in higher orders, more pairs are present). This fermion "vacuum" comes equipped with built-in fluctuations.

4f. Model with Reduction

We have seen that, without reduction, it is possible to define a stable normalizable "vacuum" state $|v\rangle$ satisfying the two "gauge" conditions $R(x)|v\rangle=0$, $\pi_s(x)|v\rangle=0$, and that the theory describes free dressed fermions. In the extreme nonrelativistic limit, the meson field dressing is an eigenstate of $R(x)$ with Yukawa-potential as eigenvalue, and this holds approximately when the fermions move. We have also considered the "vacuum" by itself when reduction is included and have seen that, although there are "vacuum" states which undergo dramatic reduction dynamics, our "gauge" restricted vacuum state $|v\rangle$ can also be made as stable as we wish with respect to the reduction process.

We shall now consider what happens to the state of a single fermion due to reduction. The density matrix evolution equation is (4.13):

$$dD(t)/dt = -i[H, D(t)] - \frac{\lambda}{2} [\int dx [R(x), L_R(x), D(t)]] \quad (4.13)$$

where $H=H_f+H_m+H_i$. First we analyze the extreme nonrelativistic limit, as we did in section 3d for the scalar model. The initial density matrix describes a pure state of a dressed fermion as in (3.35)

$$D(0) = |\int dx f(x) \tilde{\xi}^+(x) |0\rangle_f |v\rangle \langle 0|_f \langle v| \int dx f^*(x') \tilde{\xi}^-(x') \quad (4.63)$$

$$\tilde{\xi}^+(x) = e^{ig \int dx' \pi_R(x') \gamma(x-x') \tilde{\xi}^+(x)} \quad (4.38)$$

To solve Eq. (4.13), we make the same ansatz as in (3.36):

$$D(t) = \int dx dx' f(x) \tilde{\xi}^+(x) |0\rangle_f |v\rangle \langle 0|_f \langle v| \int dx' f^*(x') \tilde{\xi}^-(x') \quad (4.64a)$$

$$D(x, x', 0) = |0\rangle \langle 0| \quad (4.64b)$$

Putting (4.64) into Eq. (4.13) and utilizing Eqs. (4.39) and (4.33) results in the dynamical equation (3.37) for D , with $\phi(y)$ replaced by $R(y)$.

The discussion in section 3d is completely relevant here. The three terms on the right hand side of Eq. (3.37) describe reduction behavior for the "vacuum" alone, reduction behavior for the fermion with rate given by Eq. (3.38)

$$\text{reduction rate} = (\lambda g^2/2) \int dy |y(x-y) \gamma(x'-y)|^2 \quad (4.65)$$

and the interaction between the "vacuum" and fermion reductions governed by

$$\lambda_g [dy [y(x-y)-y(x'-y)]] [R(y), D(x, x', t)] \quad (4.66)$$

When we perform a modal analysis as was done in section 4b for the vacuum alone, we obtain e_g as a replacement for Eq. (4.26)

$$<Q, \theta | \tilde{D}_c(t) Q, \theta > = f(Q) f^*(Q) e^{-\lambda_1 t} e^{-(\lambda_1/4\epsilon)(Q - Q')^2}.$$

$$\begin{aligned} & \cdot e^{(\lambda_1/8\epsilon^2)(Q^2 \sin^2(\theta-\epsilon t) - \sin^2\theta) + Q^2 [\sin^2(\theta-\epsilon t) - \sin^2\theta] - 2QQ' [\sin(\theta+\theta'-2\epsilon t) - \sin(\theta+\theta')]} \\ & \cdot e^{(\lambda_2/\epsilon) [Q [\sin(\theta-\epsilon t) - \sin\theta] - Q' [\sin(\theta'-\epsilon t) - \sin\theta']}] \quad (4.67) \end{aligned}$$

(λ_1 and λ_2 are defined in Eq. (3.41): the cosine terms there are to be associated with $\tilde{D}_c$ and the sine terms with $\tilde{D}_r$).

We see from Eq. (4.67), just as for the scalar model (section 3d, Eq. (3.42)), that the major features of the reduction are the reduction of the fermion states due to the Yukawa-potential as expressed by the $\exp^{-\lambda_1 t}$ term, the reduction of the "vacuum" described by the exponents $\sim \lambda$ (the same expression as in Eq. (4.26)), and the last term that corresponds to the fermion-"vacuum" interaction (4.66). This last term just oscillates with frequency ϵ . The exponent is as small as we wish because it is proportional to Q and, for our choice of "vacuum," Q is less than an arbitrary small value beyond which $f(Q)$ vanishes. Thus there is no dramatic new behavior for the "vacuum" modes when we add fermions in the extreme nonrelativistic limit: the "vacuum" state remains $|v>$, and the fermion state reduces satisfactorily.

However, when we allow the fermion to move, it may very well be a different story. If, instead of the extreme nonrelativistic approximation (4.38) for $\tilde{\xi}_t^+(x)$, we use the nonrelativistic expression (4.50), then $\tilde{\xi}_t^+(x)$ is no longer an eigenstate of $R(y)$. Making the ansatz

$$D(t) = \int dx dx' \tilde{\xi}_t^+(x) |0>_t D(x, x', t) \int dx' \tilde{\xi}_t^-(x'), \quad D(x, x', 0) = f(x) |v> < 0 | f^*(x') \quad (4.68)$$

in order to solve Eq. (4.13), and employing Eq. (4.48) to commute H past $\tilde{\xi}_t^+(x)$, with use of the expression (4.50) for $\tilde{\xi}_t^+(x)$, the dynamical equation for D is

$$\begin{aligned} dD(x, x', t)/dt = & -i[H_m, D] + i \frac{1}{2M} (\nabla^2 - \nabla'^2) D' \\ & - \frac{\lambda}{2} [dy \{ \bar{R}(x, y)^2 D + D \bar{R}(x', y)^2 - 2 \bar{R}(x, y)^2 D \bar{R}(x', y)^2 \}] \quad (4.69) \end{aligned}$$

where $\bar{R}(y)$ and the first few terms of its expansion in powers of g is given by

$$\begin{aligned} \bar{R}(x, y) = & T_R \{ \exp[i \int_0^\infty dt e^{-\delta t'} e^{-it'(2M)-1V^2} \tilde{S}(x, t') e^{it'(2M)-1V^2}] R(y) \cdot \\ & \cdot T \{ \exp[-i \int_0^\infty dt e^{-\delta t'} e^{-it'(2M)-1V^2} \tilde{S}(x, t') e^{it'(2M)-1V^2}] \} \\ = & R(y) + i g \int_0^\infty dt_1 e^{-\delta t_1} e^{-it_1(2M)-1V^2} G(x-y, t_1) e^{it_1(2M)-1V^2} + \\ & + g^2 \int_0^\infty dt_1 \int_0^\infty dt_2 e^{-\delta(t_1+t_2)} e^{-it_1(2M)-1V^2} G(x-y, t_1) e^{it_1(2M)-1V^2} \\ & \cdot e^{-it_2(2M)-1V^2} \tilde{S}(x, t_2) e^{it_2(2M)-1V^2} + \dots \quad (4.70) \end{aligned}$$

(T_R is the time reversed operator, and $\tilde{S}$ is given in Eq. (4.36).)

It is apparent from Eq. (4.70) that in second and higher orders of g , $\bar{R}(y)$ involves, through $\tilde{S}$, the operators $\pi_R(x)$ and $S(x)$. These operators could, in a significant way, send the "vacuum" out of the gauge invariant state $|v>$. In the modal language of sections 4a and 4b, π_R and S depend upon the momentum operators P_i and s_i (see Eqs. (4.9b), (4.11a) and (4.17)). Now, the wavepackets that describe the "vacuum" state $|v>$ for each mode have been carefully constructed to be highly localized in Q_i and η_i about the origin. Momentum dependent terms in the evolution equation can be expected to expand those wavepackets. Then the "gauge" restrictions no longer hold and all hell breaks loose because the "vacuum" gets caught up in the reduction dynamics as it tries to reduce to a random sample field of $R(y)$ (section 4b, Eq. (4.24)). It may be that a dynamic "vacuum" is more appropriate for this theory than a static one. How, or whether the theory makes sense in these circumstances is yet to be seen.

This situation has not yet been analyzed, and it is of great interest. One of the things that makes it interesting is something we have foregone mentioning until now. We have shown (in Eq. (4.14)) that the expectation value of the energy $\bar{H}$ does not change. But we know that fermion states are reducing and, therefore, if the physical picture introduced by GRW is valid here, reducing wavepackets are narrowed and therefore suffer an energy increase (Eq. (2.13)). So, why does $\bar{H}$ remain constant? Are the wavepackets in fact narrowed, with an accompanying decrease of energy by the "vacuum" to maintain the energy balance? Or, might the "vacuum" act back upon a wavepacket, to spread it in a compensating manner, even as it is being reduced so that the particle suffers no energy increase due to reduction? At this time we do not know what happens.

4g. Numerical Considerations

We hope we have demonstrated that the physical picture of reduction of fermion states based upon an attached Yukawa-potential "handle"²⁰ is worth considering, along with the GRW¹³⁻¹⁶ picture of reduction on particle position or its CSL generalization¹⁷ of reduction on particle number density. The physical mechanism of "handle" reduction is different from that of GRW or CSL, and new and interesting features arise. One thing we have not yet done here, which has been done for the latter two models,²⁸ is to consider the parameter choices and the experimental situation.

There are two considerations that are involved. One is that the reduction rate be "fast" for a macroscopic object and "slow" for a microscopic object. The other is that the energy change suffered by individual particles due to the reduction process be within experimental bounds. But, since as remarked at the end of the last section, we do not yet know what the theory predicts about energy change, such discussion as we can provide should be considered provisional (but see below).

As we have shown, the off-diagonal elements of the density matrix, taken between position eigenstates, for a single particle in the extreme nonrelativistic limit, decay at the rate given by Eq. (4.65) :

$$\text{single particle reduction rate} = (\lambda g^2/2) \frac{1}{4\pi m} [1 - e^{-m k \cdot x}] \quad (4.71)$$

This may be compared with the GRW rate $\lambda_{\text{GRW}}(1 - \exp(-(\alpha/4)(k \cdot x)^2))$ given in Eq. (2.12b), where $\lambda_{\text{GRW}} \approx 10^{-16} \text{sec}^{-1} \approx 10^{-8} \gamma \tau^{-1}$ and $\alpha^{-1/2} \approx 10^{-5} \text{cm}$. If we wish, we may choose

$$\lambda g^2/m \approx \lambda_{\text{GRW}}, \quad m^{-1} \approx \alpha^{-1/2} \quad (4.72)$$

to emulate the GRW choices of a reduction rate that is constant for distances $\geq \alpha^{-1/2}$ at a value λ_{GRW} .

However, it should be pointed out that the GRW parameters were chosen partly based upon energy production considerations, which may not hold here. The direct experimental upper bound on the reduction rate for fermions is surprisingly high, and consistent with a much more rapid rate than $10^{-16}/\text{sec}$. To the best of our knowledge, this was obtained by Zeilinger⁶, who observed a two-slit neutron interference pattern (obtained at a distance of 5 m. from the slits, with slit separation $1.260 \times 10^{-5} \text{cm}$). This pattern would have deviated from its agreement with the quantum theory prediction if an assumed constant reduction rate for the duration of the neutron's flight was larger than .1/sec. (Actually, the neutron packets overlapped at about .5m. from

the slits due to diffraction of the 18.5 Angstrom wavelength neutrons by the $220 \times 10^{-5} \text{cm}$. width slits, so the limit should perhaps be closer to 1/sec.)

The reduction rate given by CSL^{18,25} for a macroscopic object, e.g. of 1 cm. dimensions and normal density, in a superposition of states where the center of mass in the two states is separated by 10^{-5}cm , is about 10^{13}sec^{-1} . It is given by the expression

$$\text{CSL macroscopic object reduction rate} \approx \lambda_{\text{GRW}} \alpha^{-3/2} D N \quad (4.73)$$

where D is the particle number density $\approx 10^{24}/\text{cm}^3$ and N is the number of particles uncovered when we imagine starting with two exactly overlapping copies of the object, and displacing one of them. (When the separation is so large that there is no particle overlap of the two states, N is the number of particles in the object.) This certainly conforms with the philosophical position that a macroscopic object should not remain in a superposition of states for a time that is perceivable by human senses ($\approx 10^{-3} \text{secs}$), but there appears to be no more stringent experimental bound.

The reduction rate for a macroscopic object in a superposition of two displaced states is also given by Eq. (4.65), if we take y to be the total Yukawa-potential due to all particles. If we imagine an object of dimensions much larger than m^{-1} , then the total Yukawa-potential at any point inside the object, due to all particles, is approximately constant at the value

$$y = \int d^3x D e^{-m|x|/4\pi|x|} = D/m^2 \quad (4.74)$$

and can be taken to be approximately zero outside the object. If a copy of the object is imagined displaced from the original, there is no difference in their Yukawa fields where the two objects overlap. The contribution to the integral in (4.65) comes from the two regions of identical volume V where they don't overlap, and where the Yukawa-potential difference between the two objects is just (4.74). We thus obtain

$$\text{macroscopic object reduction rate} \approx (\lambda g^2/2)(D/m^2)2V = \lambda g^2/m^4 D N \quad (4.75)$$

Comparison with Eq. (4.73) shows that we may obtain agreement with the CSL prediction provided

$$\lambda g^2/m^4 \approx \lambda_{\text{GRW}} \alpha^{-3/2} \quad (4.76)$$

The choices (4.72) are consistent with (4.76).

5. Quantum Electrodynamics with Reduction

The two-meson-field model in section 4 was constructed to bear a strong relation to the structure of the longitudinal and timelike photon sector of QED. We shall see that many features of the model can be carried over to QED. Moreover, in section 4f we saw that the "gauge" conditions in the two-meson-field model are probably violated when fermions move during reduction dynamics, leading to evolution of the "vacuum" state. This can occur because the "gauge" conditions there are not truly like the gauge conditions of QED which one might hope would absolutely forbid such an evolution.

One important feature of QED is that there are particles carrying both signs of the charge inside ordinary matter, while in the model of section 4, because there is no identification of the mesons with any known particles, one is free to chose all fermions coupled to the mesons as having the same sign of the g-charge. I have elsewhere argued²⁰ that we should, for this reason, prefer a reducing theory based on gravity rather than electromagnetism, since the electric potential would average out to zero inside ordinary matter, and therefore not provide as effective a "handle" for reduction as the gravitational potential. Nonetheless, the effects of reduction in the context of this specific model deserve a careful examination. In what follows, we will only lay out the structure of the theory.

To carry over the ideas of relativistic CSL to QED, one must have a scalar field $R(x)$ to couple to the white noise. Furthermore, the dressed fermion state ought to be approximately an eigenvector of $R(x)$.

Following Dirac²⁴, as we fervently shall do, we perform our calculations in the Coulomb gauge where the theory can be expressed in terms of already partially dressed fermion operators

$$\psi(x)^\dagger \rightarrow \tilde{\psi}(x)^\dagger \equiv e^{igV(x)}\psi(x)^\dagger \quad (5.1)$$

($g=+e$ for a proton and $-e$ for an electron). In Eq. (5.1), the longitudinal potential $V(x)$ is related to the vector potential by $A(x) = A_{TR}(x) + \nabla V(x)$ (with $\nabla \cdot A_{TR}(x)=0$), and

$$V(x) = (2\pi)^{-3/2}(-i\sqrt{2}) \int \frac{d^3k}{g^3} [k \cdot a(k)e^{ik \cdot x} + k \cdot a(k)^\dagger e^{-ik \cdot x}] \quad (5.2)$$

In this section, we are using the nonvanishing commutation relations

$$[a(k), a(k')^\dagger] = \epsilon \delta(k \cdot k'), \quad [a^0(k), a^0(k')^\dagger] = -\epsilon \delta(k \cdot k') \quad (5.3)$$

We suggest the following candidate for the role of $R(x)$:

$$\begin{aligned} R(x) &\equiv 2^{-1/2} \partial_\mu [A_\mu^+ - A_\mu^-] \\ &= (2\pi)^{-3/2}(-i\sqrt{2}) \int \frac{d^3k}{2\epsilon} [(ea^0(k) - k \cdot a(k))e^{ik \cdot x} + (ea^0(k) - k \cdot a(k)^\dagger)e^{-ik \cdot x}] \end{aligned} \quad (5.4)$$

It is clearly a Lorentz scalar, and

$$[R(x), V(x')] = (-i\sqrt{2}) (2\pi)^{-3/2} \int \frac{d^3k}{e} \cos k \cdot (x - x') = \frac{-i}{2\sqrt{2}\pi^2} \frac{1}{(x - x')^2} \quad (5.5a,b)$$

$$[R(x), e^{igV(x')}\psi(x')^\dagger] = e^{igV(x')}\psi(x')^\dagger \frac{g}{2\sqrt{2}\pi^2} \frac{1}{(x - x')^2} \quad (5.6)$$

It follows from Eq. (5.6) that, if $R(x)|v\rangle=0$,

$$R(x)\tilde{\psi}(x)^\dagger \cdot \tilde{\psi}(x_N)^\dagger |v\rangle = \frac{g}{2\sqrt{2}\pi^2} \sum_n \frac{1}{(x - x_n)^2} \tilde{\psi}(x)^\dagger \cdot \tilde{\psi}(x_N)^\dagger |v\rangle \quad (5.7)$$

Interestingly, it is not the Coulomb potential but essentially its square (however, with the associated charge sign) that is the "handle" for reduction. While this is so singular at the origin that it makes the reduction rate integral (e.g. the exponent in Eq. (3.38)) diverge, we expect that it will be regularized at the origin over the distance M^{-1} by the further dressing of the fermion due to its motion, as in Eq. (4.53). We return to this point in some speculative remarks at the end of this section.

It is also worth noting, if we construct the electric field $E=-\nabla A^0 \cdot \hat{A}$, that

$$\nabla \cdot E = -\nabla^2(A^0 + V) = 2^{1/2} \sqrt{-\nabla^2} R(x) \quad (5.8)$$

We also define

$$\begin{aligned} \pi_S(x) &\equiv 2^{-1/2} \sqrt{-\nabla^2} \partial_\mu A^\mu \\ &= -i(2\pi)^{-3/2} \int \frac{d^3k}{2} [(ea^0(k) - k \cdot a(k))e^{ik \cdot x} - (ea^0(k) - k \cdot a(k)^\dagger)e^{-ik \cdot x}] \end{aligned} \quad (5.9a,b)$$

as well as the conjugate variables to $R(\mathbf{x})$ and $\pi_S(\mathbf{x})$:

$$\pi_R(\mathbf{x}) \equiv i(2\pi)^{-3/2} \int \frac{d\mathbf{k}}{2\epsilon^2} [(e\mathbf{a}^0 + \mathbf{k} \cdot \mathbf{a}) e^{i\mathbf{k} \cdot \mathbf{x}} - (e\mathbf{a}^0 + \mathbf{k} \cdot \mathbf{a})^\dagger e^{-i\mathbf{k} \cdot \mathbf{x}}] \quad (5.10)$$

$$S(\mathbf{x}) \equiv - (2\pi)^{-3/2} \int \frac{d\mathbf{k}}{2\epsilon^3} [(e\mathbf{a}^0 + \mathbf{k} \cdot \mathbf{a}) e^{i\mathbf{k} \cdot \mathbf{x}} + (e\mathbf{a}^0 + \mathbf{k} \cdot \mathbf{a})^\dagger e^{-i\mathbf{k} \cdot \mathbf{x}}] \quad (5.11)$$

It may be directly checked that these operators satisfy the commutation relations (4.4), with the rest of the commutators vanishing.

The complete set of operators in the longitudinal-timelike sector of QED may be expressed in terms of these operators, which also are a complete set:

$$A^0 = 2^{-1/2} [(-\nabla^2)^{-1/2} R - (-\nabla^2)^{1/2} S] \quad (5.12a)$$

$$\pi^0 = 2^{-1/2} [(-\nabla^2)^{1/2} \pi_R + (-\nabla^2)^{-1/2} \pi_S] \quad (5.12b)$$

$$V = -2^{-1/2} [(-\nabla^2)^{-1/2} \pi_R + (-\nabla^2)^{-3/2} \pi_S] \quad (5.12c)$$

$$\dot{V} = 2^{-1/2} [(-\nabla^2)^{-1/2} R + (-\nabla^2)^{1/2} S] \quad (5.12d)$$

We may now look at the gauge conditions and the form of the Hamiltonian when expressed in terms of the operators R , π_R , S , π_S .

All physical states must satisfy the gauge conditions²⁴

$$\partial_\mu A^\mu(\mathbf{x})|\phi\rangle = 0, \quad (\nabla \cdot \mathbf{E} - j^0)|\phi\rangle = 0 \quad (5.13a,b)$$

where

$$j^0 \equiv g\psi^\dagger\psi, \quad \mathbf{j} = g\psi^\dagger\boldsymbol{\gamma}\psi \quad (5.14a,b)$$

(ψ and $\psi^\dagger$ can be replaced by the physical variables $\tilde{\psi}^\dagger$ and $\tilde{\psi}$ without affecting these equations.)

We see from Eqs. (5.8), (5.9a) that the gauge conditions (5.13) are equivalent to

$$2^{-1/2} (-\nabla^2)^{-1/2} \pi_S(\mathbf{x}) |\phi\rangle = 0 \quad \text{or} \quad \pi_S(\mathbf{x}) |\phi\rangle = 0 \quad (5.15a)$$

$$[2^{1/2} \sqrt{-\nabla^2} R(\mathbf{x}) - j^0] |\phi\rangle = 0 \quad (5.15b)$$

We first remark that if we choose the general state in Eq. (5.15) to be the "vacuum" state, $|\phi\rangle = |v\rangle$, we have the "gauge" conditions $\pi_S|v\rangle = 0$, $R|v\rangle = 0$ of the two-scalar model! But in the latter model we did not have "gauge" conditions for all physical states as we have here.

As an exercise, we show in terms of these variables that the states in Eq. (5.7) satisfy the gauge conditions. From Eq. (5.12c) it is obvious that $[\pi_S, V] = 0$, so $[\pi_S, \tilde{\psi}(\mathbf{x})^\dagger] = 0$ and Eq. (5.15a) is satisfied. To see that Eq. (5.15b) is satisfied, we observe that

$$j^0(\mathbf{x}) \tilde{\psi}^\dagger(\mathbf{x}_1) \dots \tilde{\psi}^\dagger(\mathbf{x}_N) |v\rangle = g \sum_n \delta(\mathbf{x} - \mathbf{x}_n) \tilde{\psi}^\dagger(\mathbf{x}_1) \dots \tilde{\psi}^\dagger(\mathbf{x}_N) |v\rangle \quad (5.16)$$

By applying $\sqrt{-\nabla^2}$ to Eq. (5.5b) we obtain

$$\sqrt{-\nabla^2} \frac{1}{2\pi^2} \frac{1}{|\mathbf{x} - \mathbf{x}'|^2} = \delta(\mathbf{x} - \mathbf{x}') \quad (5.17)$$

Therefore, application of $2^{1/2} \sqrt{-\nabla^2}$ to Eq. (5.7) produces

$$2^{1/2} \sqrt{-\nabla^2} R(\mathbf{x}) \tilde{\psi}(\mathbf{x}_1)^\dagger \dots \tilde{\psi}(\mathbf{x}_N)^\dagger |v\rangle = g \sum_n \delta(\mathbf{x} - \mathbf{x}_n) \tilde{\psi}(\mathbf{x}_1)^\dagger \dots \tilde{\psi}(\mathbf{x}_N)^\dagger |v\rangle \quad (5.18)$$

and the desired result follows from the equality of Eqs. (5.16), (5.18).

If $R(\mathbf{x})$ acts upon a state that satisfies the gauge conditions, the result is a state that also satisfies the gauge conditions, since $R(\mathbf{x})$ commutes with all operators in Eqs. (5.15). Since $R(\mathbf{x})$ is the sole operator involved when we add the reducing terms to the Schrödinger equation, that means that the reduction process does not violate the gauge conditions.

In the Coulomb gauge, the Hamiltonian for QED is²⁴

$$H = \int d\mathbf{x} [\tilde{\psi}(-i\boldsymbol{\gamma} \cdot \nabla + M) \tilde{\psi} - \mathbf{E} \cdot \mathbf{A} + \pi_R + H_{\text{FR}}] \\ + \frac{1}{2} \int d\mathbf{x} [\nabla \cdot \dot{\mathbf{V}} - \dot{\mathbf{V}} \cdot \nabla + (\nabla^2 V)^2 - \pi^0{}^2 - \nabla A^0 \cdot \nabla A^0] + \int d\mathbf{x} j^0 A^0 \quad (5.19)$$

where H_{TR} is the transverse photon Hamiltonian, and the interaction term $-g\mathbf{j}\cdot\mathbf{V}$ has been absorbed into the fermion part of the Hamiltonian by use of $\tilde{\psi}$. We want to concentrate on the longitudinal-time-like part of the Hamiltonian (which we shall call H_{LT}) in the second line of Eq. (5.19).

When we replace all operators in Eq. (5.19) by means of Eqs. (5.12), and perform some integrations by parts, the result is

$$H_{LT} = \int d\mathbf{x} [\pi_R \pi_S + \mathbf{V} \cdot \nabla \mathbf{R}] + \int d\mathbf{x} j^0 \frac{1}{\sqrt{2}} [(-\nabla^2)^{-1/2} R - (-\nabla^2)^{1/2} S] \quad (5.20)$$

The Hamiltonian for the free fields is identical to that of the two-scalar model! (However, the interaction term is quite different, especially in that it depends upon $\mathbf{R}$ as well as S , so that S is not a free field in QED.) In particular, the discussion of the construction of the "vacuum" state $|\nu\rangle$ in section 4b is completely applicable to the QED vacuum.

As is well known, the last term in Eq. (5.20) can be combined with the first, leaving only the Coulomb interaction term

$$H_{LT} = \frac{1}{2} \int d\mathbf{x} j^0 (-\nabla^2)^{-1} j^0 = \frac{1}{2} \int d^3\mathbf{x} j^0(\mathbf{x}) j^0(\mathbf{x}') \frac{1}{4\pi|\mathbf{x} - \mathbf{x}'|} \quad (5.21)$$

(utilizing $(-\nabla^2)^{-1} j^0(\mathbf{x}) = \int d\mathbf{x}' j^0(\mathbf{x}')/4\pi|\mathbf{x} - \mathbf{x}'|$). To see how this comes about with the variables we are using, we write the last term of (5.20) that depends upon $\mathbf{R}$ as

$$\int d\mathbf{x} j^0 \frac{1}{\sqrt{2}} [(-\nabla^2)^{-1/2} R = \frac{1}{2} \int d\mathbf{x}' (-\nabla'^2)^{-1} j^0 \nabla' \cdot (-\nabla^2)^{1/2} R = \frac{1}{2} \int d\mathbf{x} j^0 (-\nabla^2)^{-1} j^0 + \frac{1}{2} \int d\mathbf{x} [(-\nabla^2)^{-1} j^0] [\nabla \cdot (-\nabla^2)^{1/2} R - j^0] \quad (5.22)$$

(The first equality follows from integration by parts, the second from adding and subtracting (5.21).

Putting Eq. (5.22) back into Eq. (5.20) yields

$$H_{LT} = \int d\mathbf{x} \{ \pi_R \pi_S + [2^{-1/2} S \sqrt{-\nabla^2} + 2^{-1} (-\nabla^2)^{1/2} j^0] [2^{1/2} \sqrt{-\nabla^2} R - j^0] \} + \frac{1}{2} \int d\mathbf{x} j^0 (-\nabla^2)^{-1} j^0 \quad (5.23)$$

Eq. (5.23) reduces to Eq. (5.21) when the Hamiltonian is applied to any physical state satisfying the gauge conditions (5.15).

Thus the dynamical equation for a statevector in QED with reduction is

$$d\phi/dt = \int d\mathbf{x} [\tilde{\psi}(-i\gamma \cdot \nabla + M)\tilde{\psi} - g\mathbf{j} \cdot \mathbf{A}_R + H_{TR}] \phi + \frac{1}{2} \int d^3\mathbf{x} j^0(\mathbf{x}) j^0(\mathbf{x}') \frac{1}{4\pi|\mathbf{x} - \mathbf{x}'|} + \int d\mathbf{x} [R(\mathbf{x}) w(\mathbf{x}, t) - \lambda R(\mathbf{x})^2] \phi \quad (5.24)$$

where all states are restricted by the gauge conditions (5.15).

The physical consequences of this theory should be investigated. An important concern is the energy increase due to reduction. Using Eq. (4.13) for the density matrix evolution it is straightforward to obtain

$$d\bar{H}/dt \equiv d\text{Tr}(\bar{H}D(t))/dt = 0 \quad (5.25a)$$

$$d\bar{H}^2/dt = (\lambda g^2/8\pi^4) \text{Tr} \int d\mathbf{x} \{ \nabla \cdot \mathbf{j} / |\mathbf{x} - \mathbf{x}'|^2 \} D(t) \quad (5.25b)$$

(For these calculations, it is simplest to use ψ instead of $\tilde{\psi}$. This makes explicit the dependence of H upon $-g\mathbf{j} \cdot \nabla \mathbf{V}$, whose commutator (5.5) with $R(\mathbf{x})$ provides the right hand side of Eq. (5.25b). Loosely speaking, Eq. (5.25b) indicates that particle motion causes an $\bar{H}^2$ increase (if $\nabla \cdot \mathbf{j} = j^0 = 0$ there is no increase). But why reduction causes no change in $\bar{H}$ is as unknown here as in the two-meson model.

Although gauge invariance works very well here, it still may be worthwhile to try abandoning it, to let the vacuum be a different state than $|\nu\rangle$ and become all churned up by the reduction process. It may be that interesting effects will appear, while the theory may respect gauge invariance in a time averaged way, thereby remaining consistent with its classical limit.

Finally, I cannot forego a speculative remark about reduction rates. Suppose the motion of a fermion regularizes the eigenvalue of $R(\mathbf{x})$, in the way mentioned at the end of section 4d, modifying $(\mathbf{x} \cdot \mathbf{x}_1)^{-2}$ in the region $|\mathbf{x} - \mathbf{x}_1| \approx M^{-1}$ so that it is finite at the origin (I emphasize that this has not yet been shown). Then the reduction rate (4.65) for a fermion of mass M in a superposition of two states at $\mathbf{x}$ and $\mathbf{x}'$ separated by a few times the Compton wavelength would be

$$\text{reduction rate} = (\lambda/2) \int d\mathbf{y} \{ g\mathbf{y}(\mathbf{y} \cdot \mathbf{x}) - g\mathbf{y}(\mathbf{y} \cdot \mathbf{x}') \}^2 - \lambda e^2 M \quad (5.26)$$

In Eq. (5.26), $\mathbf{y}$ is the modification of $(\mathbf{x} \cdot \mathbf{x}_1)^{-2}$ just referred to, with $|\mathbf{x} \cdot \mathbf{x}_1| \sim M$: the major portion of the Coulomb "handle" comes from the region in the neighborhood of a few Compton wavelengths of the particle. Eq. (5.26) is qualitatively different from our

previous expressions for the reduction rate because it is proportional to the mass of the fermion. If the fermion is a proton and we equate (5.26) to $V_{GRW} = 10^{-16} \text{ sec.}^{-1}$, we get for the dimensionless coupling constant $\lambda = 10^{-38}$.

For a macroscopic object in a superposition of two displaced states, the reduction rate is

$$\text{reduction rate} = (\lambda e^2/2) \left[\sum_n \{y(\mathbf{r} - \mathbf{r}_n) - y(\mathbf{r} - \mathbf{r}'_n)\}^2 \right] \quad (5.27)$$

The major contribution to the integral in Eq. (5.27) will come from the protons because of their larger mass, so we may neglect the contribution of the electrons. When the displacement is a few nuclear diameters or more, it is likely that it is only the Z protons of each atom, lumped in a nucleus, which have approximately overlapping squared potentials, so we obtain the sum $\sum_n y(\mathbf{r} - \mathbf{r}_n) \approx Z y(\mathbf{r} - \mathbf{r}_{\text{nuc}})$ for the protons of a single nucleus. The other cross terms in Eq. (5.27) make a negligible contribution. We thus obtain, for matter in bulk, the

$$\text{reduction rate} \approx \lambda Z^2 e^2 N M \approx \lambda Z e^2 M \quad (5.28)$$

where N is the number of atoms in the object, and M is the total mass of the object. For an object of atomic number ≈ 30 containing about 10^{23} particles, with the choice $\lambda = 10^{-38}$, the characteristic reduction time would be $\approx 10^{-9}$ secs.

Acknowledgments

I am deeply grateful for the stimulation, encouragement and support provided by GianCarlo Ghirardi and Renata Grassi. I also wish to express my sense of loss at not being able to talk about these ideas with John Bell. I would like to acknowledge the hospitality of Prof. Abdus Salam, the International Atomic Energy Agency and UNESCO, as well as financial support from Hamilton College, the Istituto Nazionale di Fisica Nucleare, and the Consorzio per l'Incremento degli Studi e delle Ricerche in Fisica dell'Universita di Trieste.

References and Remarks

1. J. S. Bell in *Sixty-Two Years of Uncertainty*, edited by A. Miller (Plenum, New York, 1990).
2. D. Bohm and J. Bub, *Reviews of Modern Physics* **38**, 453 (1966).
3. P. Pearle, *Physical Review D* **13**, 857 (1976).
4. P. Pearle, *International Journal of Theoretical Physics* **48**, 489 (1979).

5. P. Pearle, *Foundations of Physics* **12**, 249 (1982).
6. A. Zeilinger, R. Gaeher, C. G. Shull and W. Treimer in *Symposium on Neutron Scattering*, edited by J. Faber Jr. (Amer. Inst. of Phys., 1984); A. Zeilinger in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986), p.16.

7. P. Pearle, *Physical Review D* **29**, 235 (1984).
8. N. Gisin, *Physical Review Letters* **52**, 1657 (1984).
9. P. Pearle, *Physical Review Letters* **53**, 1775 (1984); *Physical Review D* **33**, 2240 (1986).
10. N. Gisin, *Physical Review Letters* **53**, 1776 (1984).
11. N. Gisin, *Helvetica Physica Acta* **62**, 363 (1989).
12. P. Pearle in *New Techniques in Quantum Measurement Theory*, edited by D. M. Greenberger (N.Y. Academy of Sci., N.Y., 1986), p.539.
13. G. C. Ghirardi, A. Rimini and T. Weber, *Physical Review D* **34**, 470 (1986).
14. J. S. Bell in *Schrödinger-Centenary Celebration of a Polymath*, edited by C. W. Kilmister (Cambridge University Press, Cambridge, 1987).
15. G. C. Ghirardi, A. Rimini and T. Weber, *Physical Review D* **36**, 3287 (1987).
16. G. C. Ghirardi, A. Rimini and T. Weber, *Foundations of Physics* **18**, 1 (1988). Also, see reference 28.
17. P. Pearle, *Physical Review A* **39**, 2277 (1989). (1990).
18. G. C. Ghirardi, A. Rimini and T. Weber, *Physical Review A* **42**, 78 (1990).
19. N. Gisin¹¹ independently obtained the general form of the nonlinear equation equivalent to the linear CSL equation. L. Diosi³¹ presented a special case, and V.P. Belavkin, *Physics Letters A* **40**, 355 (1989) arrived at both the linear and nonlinear forms; however, unlike Gisin, these authors were not working in the context of dynamical statevector reduction but were instead modeling continuous nondemolition measurement situations.
20. P. Pearle in *Sixty-Two Years of Uncertainty*, edited by A. Miller (Plenum, N.Y., 1990), p.193.
21. G. C. Ghirardi, R. Grassi and P. Pearle, *Foundations of Physics* **20**, 1271 (1990).
22. G. C. Ghirardi, R. Grassi and P. Pearle, "Relativistic Dynamical Reduction Models and Locality," to be published in the *Proceedings of the Symposium on the Foundations of Modern Physics 1990*, edited by P. Lahi and P. Mittelstaedt (World Scientific, Singapore).

23. G. C. Ghirardi and P. Pearle, "Statevector Reduction I and II", to be published in Vol. 2 of the Proceedings of the Philosophy of Science Association 1990, edited by A. Fine, M. Forbes and L. Wessels (PSA Association, Mich.).
24. P. A. M. Dirac, *The Principles of Quantum Mechanics*, 4th Ed. (Clarendon Press, Oxford, 1958), Chap. 12; *Lectures on Quantum Field Theory* (Yeshiva University, N.Y. 1966).
25. F. Karolyhazy, II Nuovo Cimento **52**, 390 (1966) and in *Sixty-Two Years of Uncertainty*, edited by A. Miller (Plenum, N.Y., 1990), p.215; F. Karolyhazy, A. Frenkel and B. Lukacz in *Physics as Natural Philosophy*, edited by A. Shimony and H. Feshbach (M.I.T. Press, Cambridge Mass., 1982), p.204 and in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986), p.109.
26. R. Penrose in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986), p. 129.
27. G. C. Ghirardi, P. Pearle and A. Rimini in *Proceedings of the 3rd Int. Symp. Foundations of Quantum Mechanics*, (Tokyo, 1989), p. 109.
28. G. C. Ghirardi and A. Rimini in *Sixty-Two Years of Uncertainty*, edited by A. Miller (Plenum, N.Y., 1990), p. 167.
29. D. Gatarek and N. Gisin in "Continuous Quantum Jumps and Infinite-Dimensional Stochastic Equations", preprint.
30. This statement requires not only Eq. (2.5), but also knowledge that the process (2.1), (2.2) does indeed lead to reduced statevectors. For example, the ensemble of statevectors $\sum_i \alpha_i |e^{i\theta_j}\rangle$, where the phases θ_j are randomly distributed has the same asymptotic density matrix $D(\infty)$ as in Eq. (2.5) without any statevector reduction.
31. L. Diosi, Physics Letters A **132**, 233 (1988).
32. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row Peterson, Illinois, 1961), Chapter 12.
33. Traditionally, one asks that the vacuum state have zero energy, momentum and angular momentum and be invariant under boosts. The momentum operator in this theory is $\mathbf{P} = \int d\mathbf{k} [Q_1 s_2 - r_2 P_1 + r_1 P_2 - Q_2 s_1]$: if the energy operator $H = \int d\mathbf{k} [Q_1 P_2 - Q_2 P_1 + r_1 s_2 - r_2 s_1]$ is thought of as generating a rotation for each mode in the 12+34 planes, then the momentum generates a rotation in the 14+23 planes. If we define $Q = p \cos \varphi$, $r = p \sin \varphi$, a joint eigenfunction of H and $\mathbf{P}$ with eigenvalues 0 is any function of φ and $\sin 2\varphi \cos(\theta - \vartheta)$: the example considered in the text can be put in this form, with $|\nu\rangle = \exp - \int d\mathbf{k} [Q(\mathbf{k})^2 + r(\mathbf{k})^2] / \sigma(\mathbf{k})^2 = \exp - \int d\mathbf{k} \rho(\mathbf{k})^2 / \sigma(\mathbf{k})^2$. If $\sigma(\mathbf{k})^2$ depends only upon $|\mathbf{k}|$ then this is a zero angular momentum state, and if σ^2 is independent of $|\mathbf{k}|$, the state is also boost invariant. However, the "gauge" conditions (4.27) require $\sigma^2(\mathbf{k} \rightarrow \infty) \rightarrow 0$,

since otherwise the integral (4.23) defining $R(\mathbf{x})$ (and likewise for $\pi_s(\mathbf{x})$) will not converge when it acts on $|\nu\rangle$, so we forego Lorentz invariance for the "vacuum" states. In view of the reduction dynamics, in which a "vacuum" state may evolve into a state which does not even have zero energy and momentum, the natural concept here seems to be the Lorentz invariance of an ensemble of possible vacuum states, rather than the Lorentz invariance of a single state $|\nu\rangle$. We emphasize that the "gauge" conditions, which are all that we use in constructing fermion states, are Lorentz invariant, even though a single state $|\nu\rangle$ is not. In this sense, the non Lorentz invariance of a "vacuum" state is unobservable.